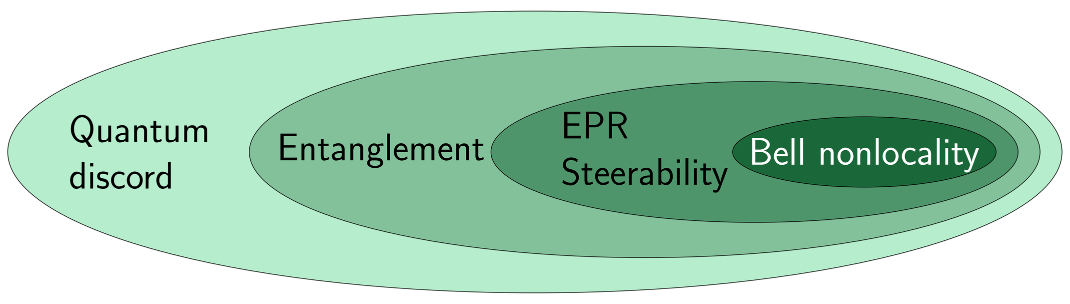
Detecting Bell correlations in a Bose-Einstein condensate

A. Braemer and **M. Gärttner**

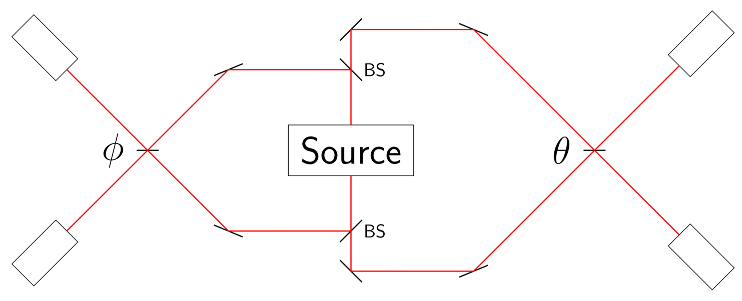
Kirchhoff-Institut für Physik, Ruprecht-Karls-Universität Heidelberg, Im Neuenheimer Feld 227, 69120 Heidelberg, Germany

Motivation

The detection of EPR correlation and genuine multipartite entanglement in our spinor BEC experiment naturally led to the question wether even stronger Bell correlation could also be witnessed.



Bell experiments with continuous variables are rather standard in quantum optics but not really established for ultra-cold atoms.



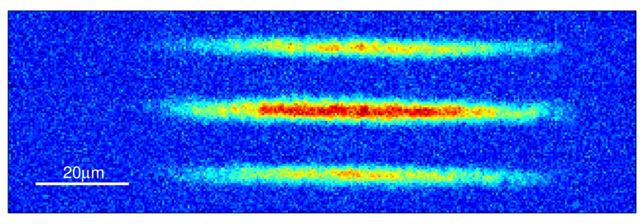
Experimental Setup

- ^{87}Rb spinor BEC
- Assumption: no spatial degrees of freedom (single mode approximation)
- Observables: atom numbers $\hat{n}_{0,\pm1}$ $(\hat{N}_{0,\pm1})$ in $F=1$ $(F=2)$
- Initial state: All atoms in $F=1, m=0$
- Entanglement source: Spin-changing collisions

$$\hat{H}_{SCC} = 2\lambda\left(\hat{a}_0^\dagger\hat{a}_0^\dagger\hat{a}_{+1}\hat{a}_{-1} + \hat{a}_0\hat{a}_0\hat{a}_{+1}^\dagger\hat{a}_{-1}^\dagger\right)$$

- For short times t: evolution under $\hat{H}_{SCC}$ changes $\hat{a}_0$-mode negligibly
  $\rightarrow$ two-mode squeezed vacuum state

$$|\psi\rangle = \exp\left[-2itN\lambda\left(\hat{a}_{+1}\hat{a}_{-1} + \hat{a}_{+1}^\dagger\hat{a}_{-1}^\dagger\right)\right]|0\rangle$$



Theory

- Standard CHSH inequality: $|E(\phi,\theta) + E(\phi',\theta) - E(\phi,\theta') + E(\phi',\theta')| \leq 2$
- Correlators:

$$E(\phi,\theta) = \frac{\left\langle\left(\hat{N}_{+1} - \hat{N}_{-1}\right)(\hat{n}_{+1} - \hat{n}_{-1})\right\rangle_{\phi,\theta}}{\left\langle\left(\hat{N}_{+1} + \hat{N}_{-1}\right)(\hat{n}_{+1} + \hat{n}_{-1})\right\rangle} \equiv \frac{\tilde{E}(\phi,\theta)}{C}$$

- Bell witness:

$$\mathcal{B} = 2C - |\tilde{E}(\phi,\theta) + \tilde{E}(\phi',\theta) - \tilde{E}(\phi,\theta') + \tilde{E}(\phi',\theta')| \geq 0$$

- Problem: sidemode populations $\hat{n}_{\pm1}$ are small

$\rightarrow$ Rewrite correlator in terms of quadratures $\hat{X} = \hat{a}^\dagger + \hat{a}$ and $\hat{P} = -i\left(\hat{a}^\dagger - \hat{a}\right)$:

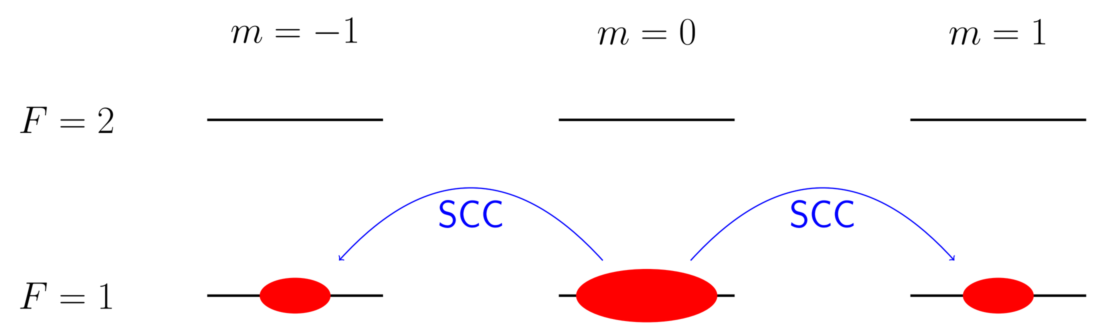
$$\hat{N}_{+1} - \hat{N}_{-1} = \hat{X}_{+1}^2 - \hat{X}_{-1}^2 + \hat{P}_{+1}^2 - \hat{P}_{-1}^2$$

$\rightarrow$ Use homodyne measurements to obtain the quadratures

Experimental protocol

Step 1
Entanglement generation

Spin-changing collisions (SCC) generate entanglement between the sidemodes. $m=\pm1$
Run the process for time t to produce a spin-squeezed state with squeezing parameter $r = 2N\lambda t$
Optics analogy: Parametric downconversion yielding 2 entangled photons

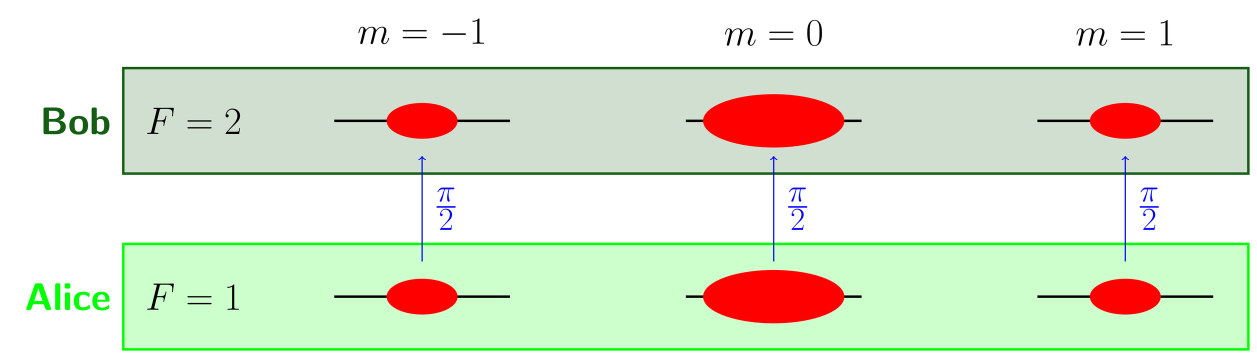


Step 2
Distribution

A $\frac{\pi}{2}$-pulse splits the atoms equally to the $F=2$ states.

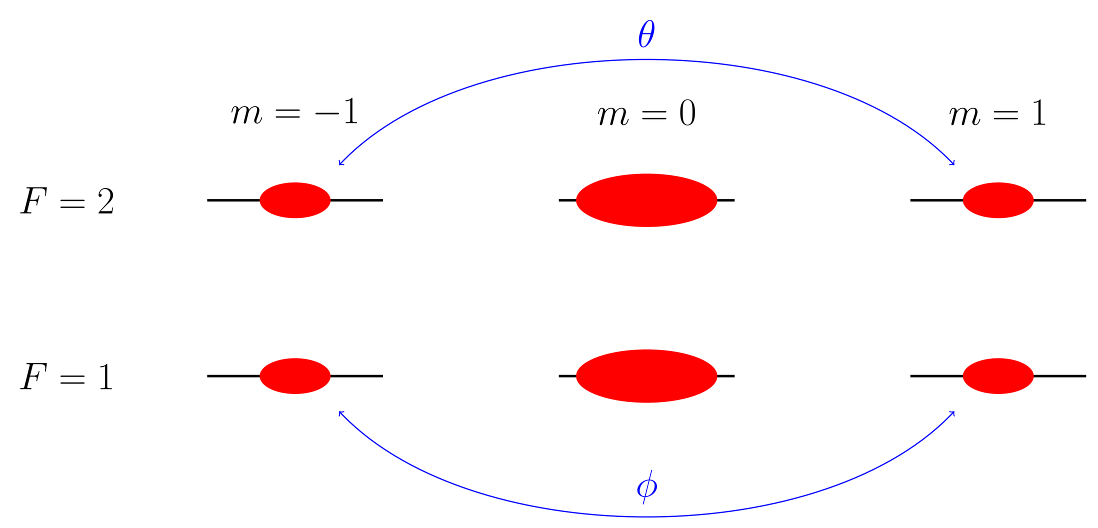
$$\hat{H} = \hat{a}_{-1}^\dagger\hat{A}_{-1} + \hat{a}_0^\dagger\hat{A}_0 + \hat{a}_{+1}^\dagger\hat{A}_{+1} + \text{h.c.}$$

Optics analogy: 50/50-Beamsplitter



Step 3
Choose measurement bases

Mix the $m=\pm1$ modes with mixture angles ϕ (θ) in $F=1$ $(F=2)$.
ϕ and θ are the typical Bell angles: $\phi=0$, $\phi'=\frac{\pi}{2}$, $\theta=\frac{\pi}{4}$, $\theta'=\frac{3\pi}{4}$
Optics analogy: Polarization filter

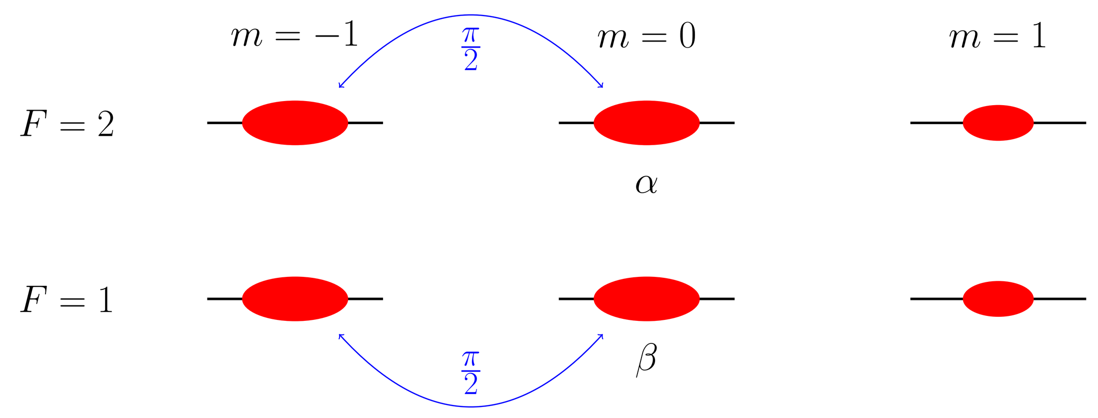


Step 4
Detection

Use homodyne detection to measure the small sidemode populations.
Homodyne detection: Mix the signal mode (f.e. $F=1, m=1$) with a local oscillator mode (in this case $F=1, m=0$) having a phase α. Then

$$\hat{X}_{+1} \propto (\hat{n}_{+1} - \hat{n}_0)|_{\alpha=0}$$
$$\hat{P}_{+1} \propto (\hat{n}_{+1} - \hat{n}_0)|_{\alpha=\frac{\pi}{2}}$$

This gives 16 combinations to measure per correlator $E(\phi,\theta)$.



Theoretical analysis

- Using undepleteded pump approximation (with squeezing parameter $r = 2\lambda N t$)

$$|\Psi_{squeezed}\rangle = \frac{1}{\cosh r}\sum_n (-i\tanh r)^n |n, N-2n, n\rangle$$

- Apply the operations backwards on $\mathcal{B}$ (Heisenberg picture)
- Evaluate the witness $\mathcal{B}$ on the squeezed state to get

$$\mathcal{B}(r) = \left(\left(2 - \frac{1}{\sqrt{2}}\right)\cosh(2r) - 1 - \frac{1}{\sqrt{2}}\right)\sinh^2(r)$$

$\rightarrow$ Violation of witness for squeezing values

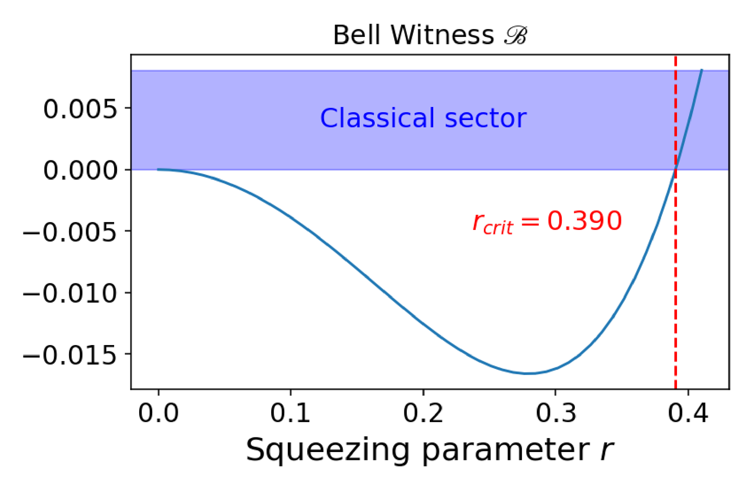
$$0 < r < r_{crit} \approx 0.390$$

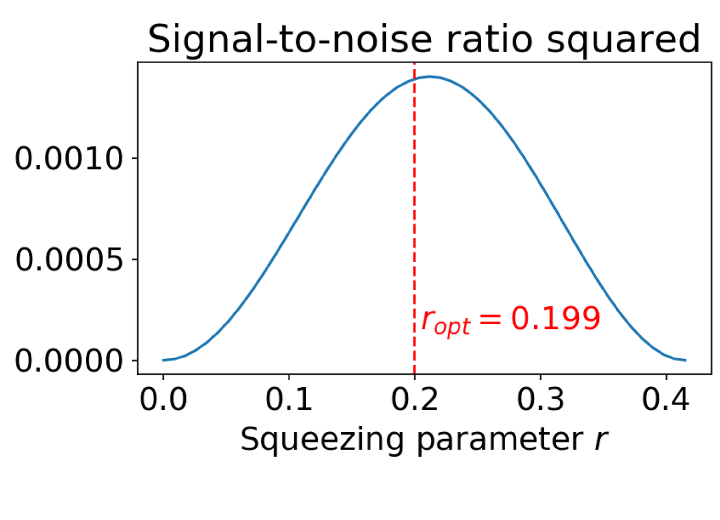
- The best signal-to-noise ratio $(SNR = \frac{\mathcal{B}}{\Delta\mathcal{B}})$ is obtained at $r_{opt} \approx 0.199$

$\rightarrow$ Need to measure whole witness about 1000 times

$\rightarrow$ Whole witness $\widehat{=}$ 5 particle number correlators each needing 16 shots (because of homodyneing)

$\rightarrow$ Means about 80.000 runs needed!





Open Questions

- Influence of various error sources:
  - Particle number fluctuation
  - Finite detector resolution
  - Technical fluctuation of different parameters (e. g. mixing angles ϕ, θ, pulse timings...)
- How to do the mixings in step 4 and 5? Raman processes?
- Possibilities to reduce the number of runs of the experiment?
  $\rightarrow$ Split BEC spatially and run multiple copies in parallel?
- More efficient read-out?
- Better witness? Other observables?

References

[1] P. Kunkel et al. **Science 360**, 6387 (2018)
[2] M. D. Reid and D. F. Walls **Phys. Rev. A 34**, 1260 (1986)