Experimental detection of out-of-time-order correlators

Martin Gärttner

Kirchhoff-Institut für Physik, Universität Heidelberg, Im Neuenheimer Feld 227, 69120 Heidelberg

Summary

Motivation:

- Out-of-time-order correlators (OTOCs) quantify operator spreading
- OTOCs are a new tool for characterizing dynamics of quantum manybody systems
- Connection to quantum gravity: Black holes are the "fastest scramblers"

Central goals:

- Devise protocols for measuring OTOCs experimentally
- Understand how OTOCs can reveal quantum chaos in semi-classical regimes
- Use OTOCs for detecting higher order correlations and entanglement

OTOCs quantify operator growth (scrambling)

Squared commutator

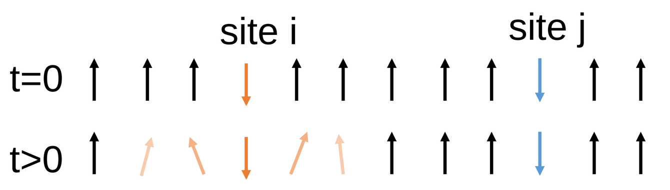
OTOC part

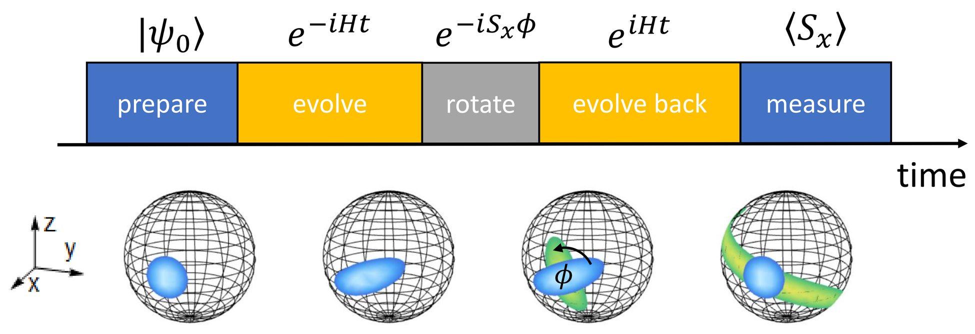
$$C(t) = \left\langle \left| [\hat{W}(t), \hat{V}] \right|^2 \right\rangle = \langle \hat{W}(t)^\dagger \hat{V}^\dagger \hat{W}(t) \hat{V} \rangle - \langle \hat{W}(t) \hat{W}(t)^\dagger \hat{V}^\dagger \hat{V} \rangle + \cdots$$

Example: $\hat{W} = \hat{\sigma}_i^x$ $\quad \hat{V} = \hat{\sigma}_j^x$ $\quad \hat{H} = \sum \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$ Ising chain, local interaction

$$\hat{W}(t) = e^{-i\hat{H}t} \hat{\sigma}_i^x e^{i\hat{H}t} = (1 - i\hat{H}t - \cdots)\hat{\sigma}_i^x (1 + i\hat{H}t + \cdots) = \hat{\sigma}_i^x + it(\hat{\sigma}_i^y \hat{\sigma}_{i+1}^z - \cdots) + \cdots$$

→ operators grow in time, quantified by commutator "scrambling"

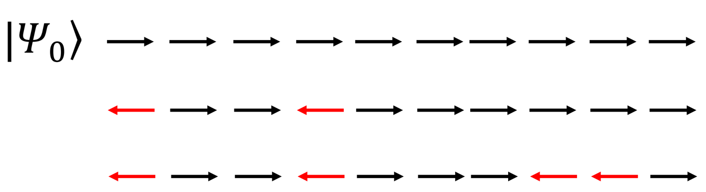




Collective dynamics

$$\langle S_x \rangle = \langle \Psi_0 | \, e^{itH} \, e^{i\phi S_x} \, e^{-itH} S_x \, e^{itH} \, e^{-i\phi S_x} \, e^{-itH} | \Psi_0 \rangle$$

$$= \sum_m \langle \Psi | C_m | \Psi \rangle \, e^{i\phi m} \qquad C_m = \sum \underbrace{\sigma_1^z \sigma_4^y \ldots \sigma_k^z}_{\text{At least m terms}} \qquad \hat{H} = \sum \hat{\sigma}_i^z \hat{\sigma}_j^z$$

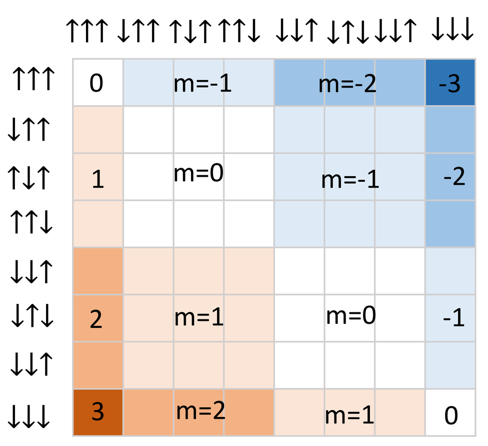


$$\sigma_i^z \sigma_j^z = \sigma_i^+ \sigma_j^+ + \sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+ + \sigma_i^- \sigma_j^-$$

→ OTOCs detect buildup of higher order correlations

OTOCs quantify coherence and entanglement

Multiple quantum coherences



$$\rho = \sum_m \rho_m \qquad I_m = Tr[\rho_m \rho_m^\dagger]$$

[NMR: Pines group 1980s]

Measure initial state overlap:

$$\langle \rho_0 \rangle = Tr[\rho_0 \rho_f] = Tr[\rho_0 \, e^{itH} e^{-i\phi S_x} \, e^{-itH} \rho_0 \, e^{itH} \, e^{i\phi S_x} \, e^{-itH}]$$

$$= Tr[\, e^{-itH} \rho_0 \, e^{itH} e^{-i\phi S_x} \, e^{-itH} \rho_0 \, e^{itH} \, e^{i\phi S_x}]$$

$$= Tr[\rho \, e^{-i\phi S_x} \rho \, e^{i\phi S_x}] \qquad \phi = 0\text{: purity}$$

$$= \sum_{m=-N}^{N} \underbrace{Tr[\rho_m \rho_m^\dagger]}_{I_m} \, e^{im\phi}$$

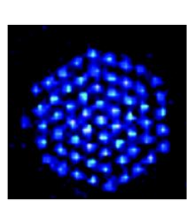
Quantum Fisher information = sensitivity of state (fidelity) with respect to a rotation:

$$F_Q(|\psi\rangle, A) = -2 \frac{d^2}{d\phi^2} \left| \langle \psi | e^{iA\phi} | \psi \rangle \right|^2_{\phi=0}$$

$$F_Q(\rho, A) = -2 \frac{d^2}{d\phi^2} \mathcal{F}(\rho, \rho_\phi) \Big|_{\phi=0} \geq -2 \frac{d^2}{d\phi^2} Tr[\rho_0 \rho_f] \Big|_{\phi=0}$$

→ Witness for multipartite entanglement

Time-reversal with trapped ions



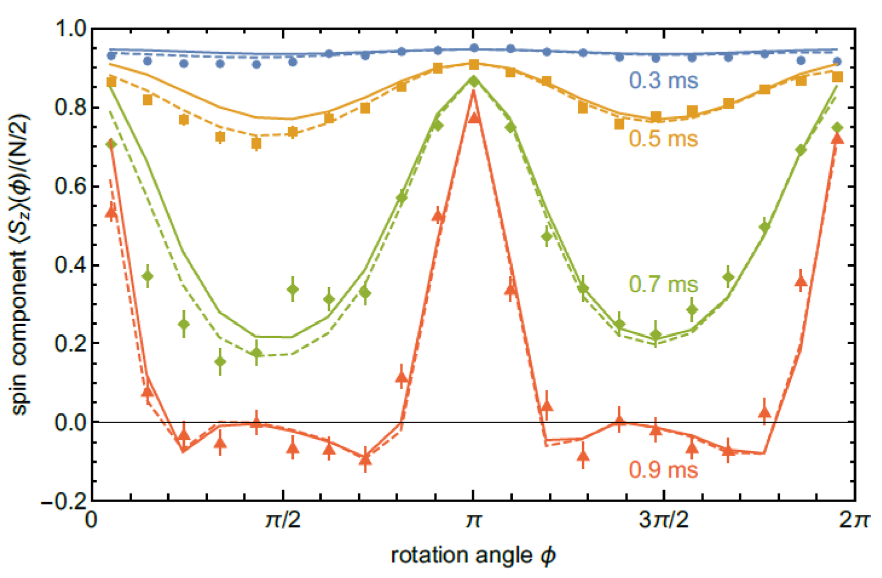
N=110 $^9Be^+$ ions (Bollinger group, NIST)

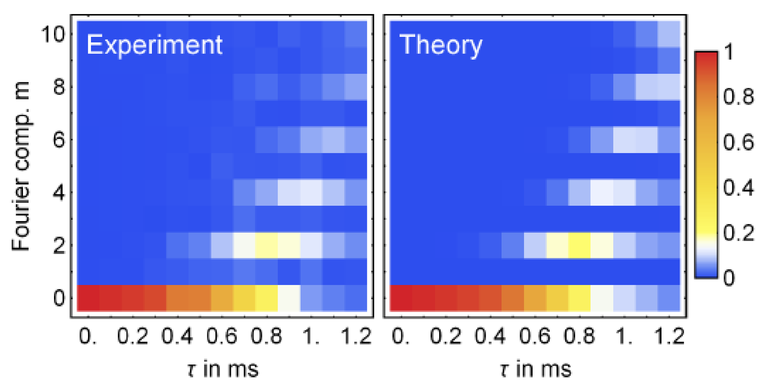
$^2S_{1/2}$ $\quad |{\uparrow}\rangle = |m = 1/2\rangle$

124 GHz

$|{\downarrow}\rangle = |m = -1/2\rangle$

- Interactions mediated by phonons, reversible
- Spin rotations: MW coupling





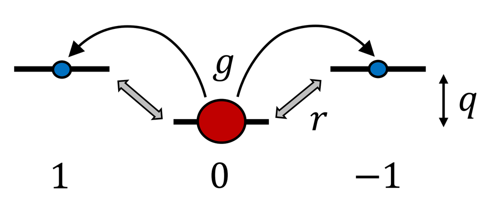
Gärttner et al. Nat. Phys. **13**, 781 (2017)

Gärttner, Hauke, Rey, Phys. Rev. Lett. **120**, 040402 (2018)

OTOCs in a three-mode bosonic system

Rautenberg and Gärttner, arXiv:1907.04094

Model Hamiltonian and Symmetries



$$H = g(a_{-1}^\dagger a_1^\dagger a_0 a_0 + a_0^\dagger a_0^\dagger a_{-1} a_1 + \cdots) + q(N_1 + N_{-1}) \quad \text{Conserves } N_1 - N_{-1}$$

$$+ \frac{r}{\sqrt{2}} \left((a_{-1}^\dagger + a_1^\dagger) a_0 + a_0^\dagger (a_{-1} + a_1) \right)$$

Classical mean field dynamics

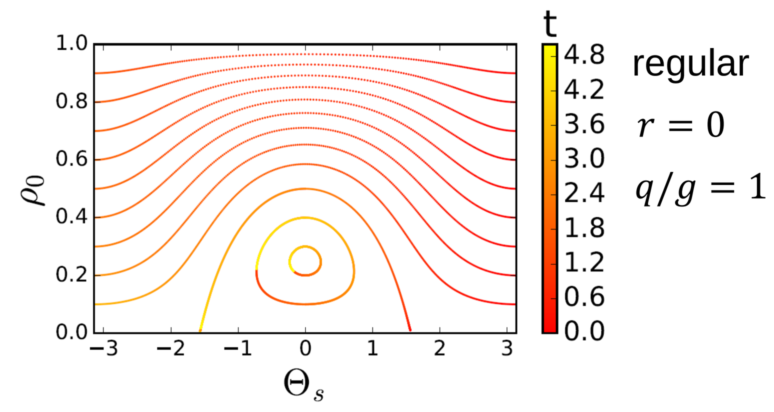
$$a_i \to \zeta_i \sqrt{N} \qquad a_i^\dagger \to \zeta_i^* \sqrt{N} \qquad \zeta_i, \zeta_i^* \to (\rho_0, \theta_s, m, \theta_m)$$

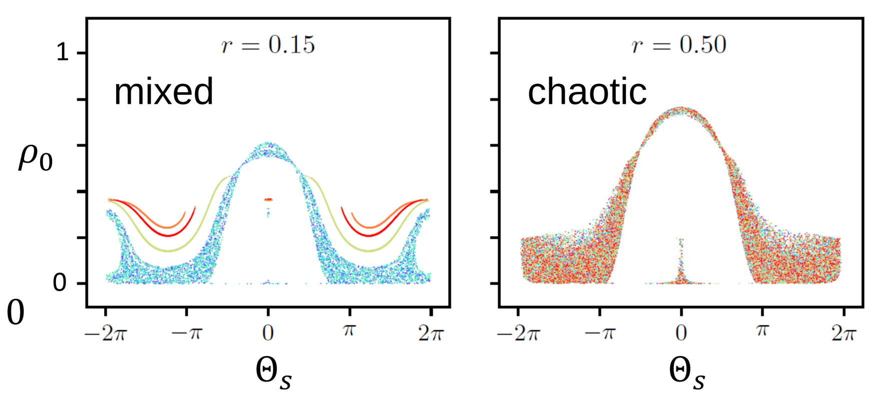
N_0/N $\qquad (N_1 - N_{-1})/N$

$$H_{\text{mf}} = gN\rho_0 \left\{ (1-\rho_0) + \sqrt{(1-\rho_0)^2 - m^2} \cos\Theta_s \right\} + \frac{gN}{2} m^2 + q(1-\rho_0)$$

Class. integrable

$$+ r\sqrt{\rho_0} \left\{ \sqrt{1-\rho_0+m} \cos\left(\frac{\Theta_s + \Theta_m}{2}\right) + \sqrt{1-\rho_0-m} \cos\left(\frac{\Theta_s - \Theta_m}{2}\right) \right\}.$$



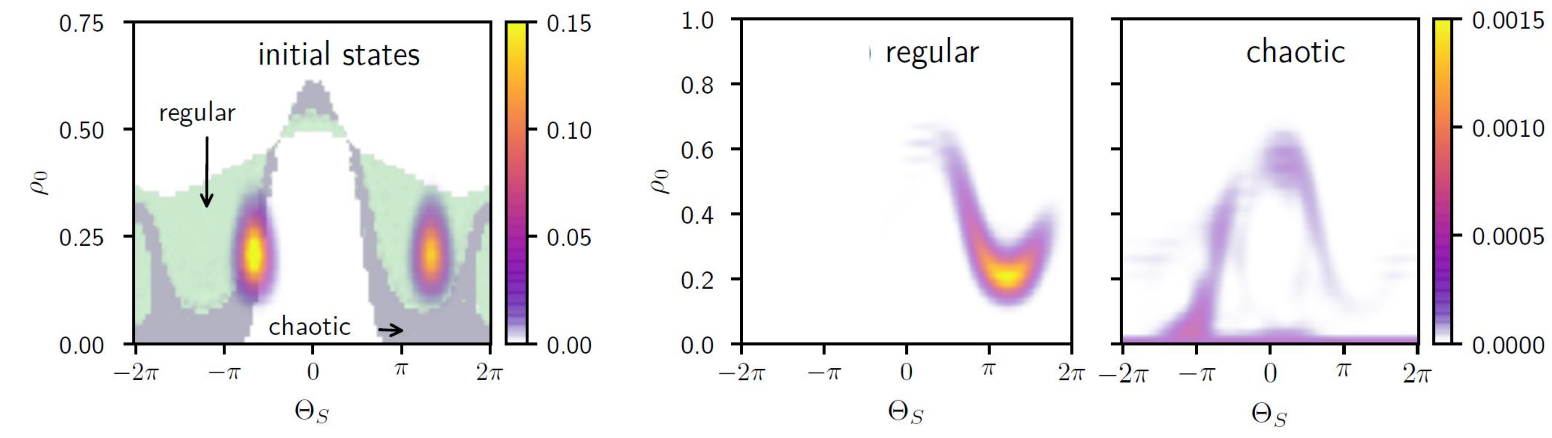


Poincaré sections at $\Theta_m = 0$

Quantum dynamics

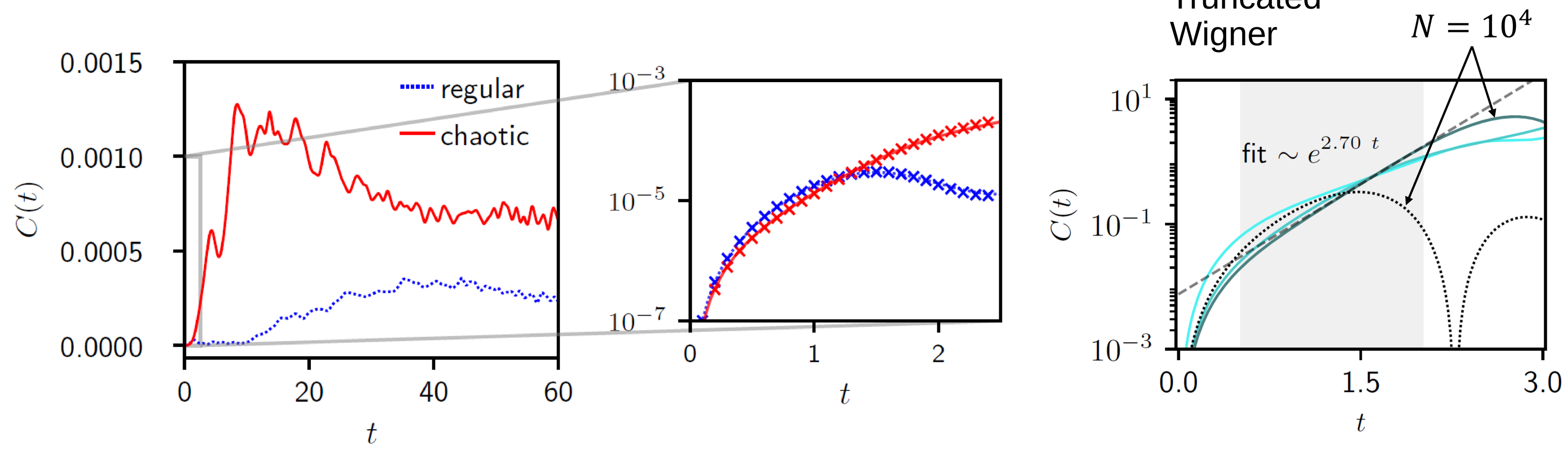
Husimi distributions

Parameters: $N = 100, r = 0.15, t = 10$



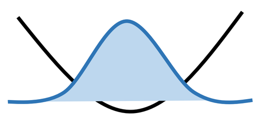
OTOCs

$$C(t) = \langle |[\hat{\rho}_0(t), \hat{\rho}_0(0)]|^2 \rangle \qquad \hat{\rho}_0 = \hat{N}_0/N$$

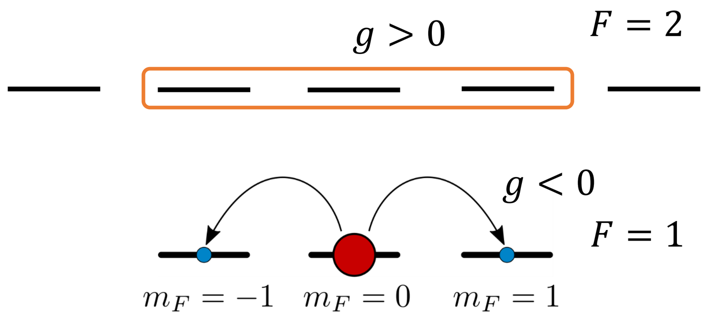


Possible experimental realization with rubidium BEC

Spin-changing collision dynamics in rubidium



Rb BEC
Single spatial mode



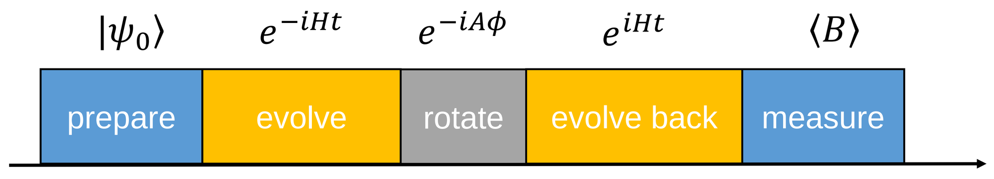
g: Spin-changing collisions opposite sign in F=2

q: Quadratic Zeeman effect (and MW dressing)

r: rf drive

q and r freely tunable

Readout protocol



Protocol yields: $\langle e^{iA(t)\phi} B e^{-iA(t)\phi} \rangle$

$$= \Lambda + \phi^2 [\langle A(t) B A(t) \rangle - \Lambda \langle A^2(t) \rangle] \equiv \Lambda + \phi^2 \Gamma_B^{(2)}$$

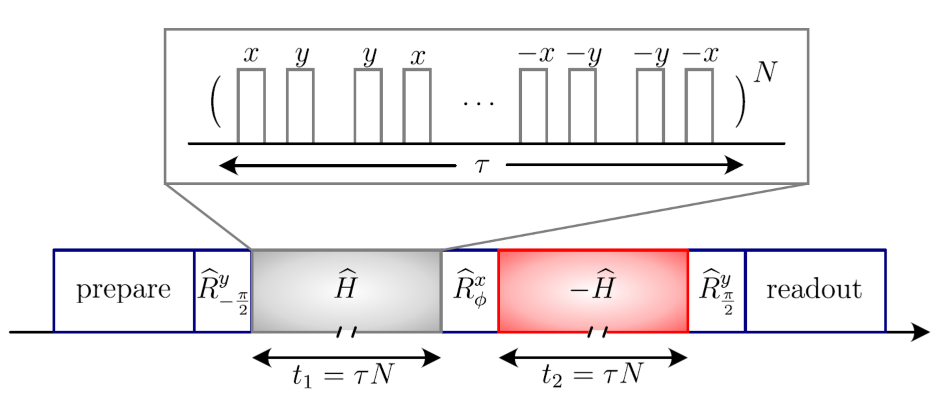
Assume $B|\psi_0\rangle = \Lambda|\psi_0\rangle$ eigenstate

An thus:

$$C(t) = \langle |[A(t), B(0)]|^2 \rangle = -2\Lambda \Gamma_B^{(2)} + \Gamma_{B^2}^{(2)}$$

Collaboration with group of M. Oberthaler

Time-reversal through Hamiltonian engineering



Similar sequence for engineering $-\hat{H}$

Implementation: Rydberg spin system.
Collaboration with group of M. Weidemüller

Preliminary results: XXX model