Variational ground state search on the BrainScaleS-2 neuromorphic hardware

15th March 2021, Virtual APS March Meeting

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BrainScaleS

STRUCTURES CLUSTER OF EXCELLENCE

Simulating many-body systems with neural quantum states

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$\uparrow\uparrow\downarrow\uparrow$

system of N qubits: $|\psi\rangle = \sum_{v_1,\ldots,v_N} c_{v_1,\ldots,v_N} |v_1 \ldots v_N\rangle$

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$|\psi_\theta\rangle$

desired variational ansatz:

- efficient representation $\dim(\theta) = \mathrm{poly}(N)$
- efficient sample generation

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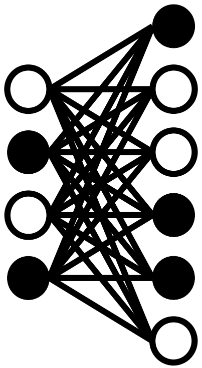

synthetic measurement distribution p_θ

expectation values $\langle O \rangle_{p_\theta}$

optimization $\Delta\theta$

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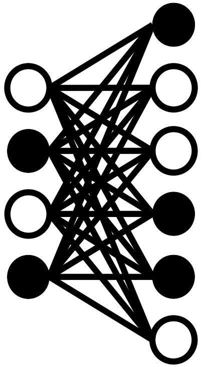
Restricted Boltzmann machine



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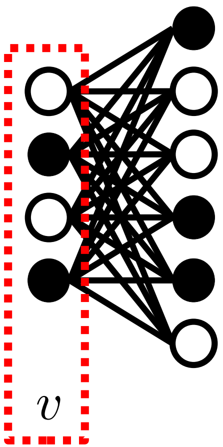

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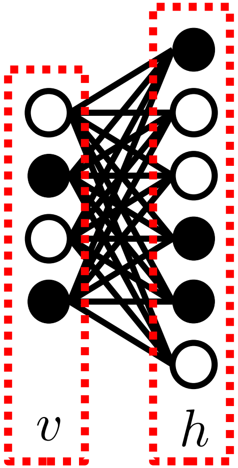

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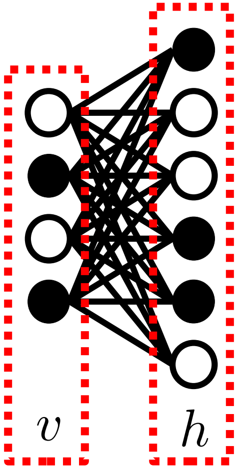
$\theta = [W, b]$

$\dim(\theta) = N_v N_h + N_v + N_h$



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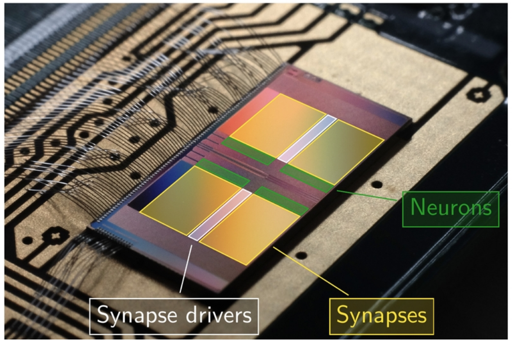
Why neuromorphic hardware for neural quantum states?

BrainScaleS-2 (BSS2)
neuromorphic chip



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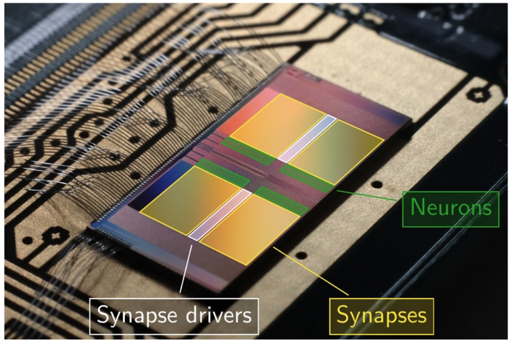
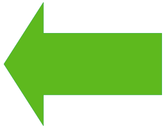
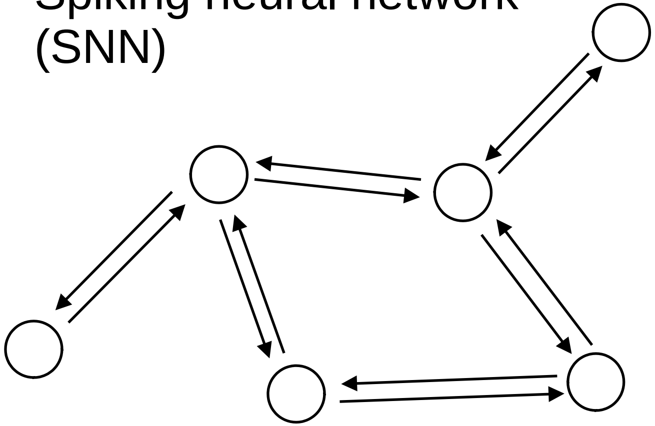




Spiking neural network (SNN)



- BSS2 implements physical SNN as continuous dynamical system

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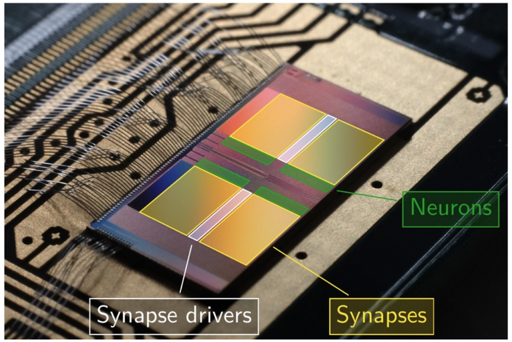

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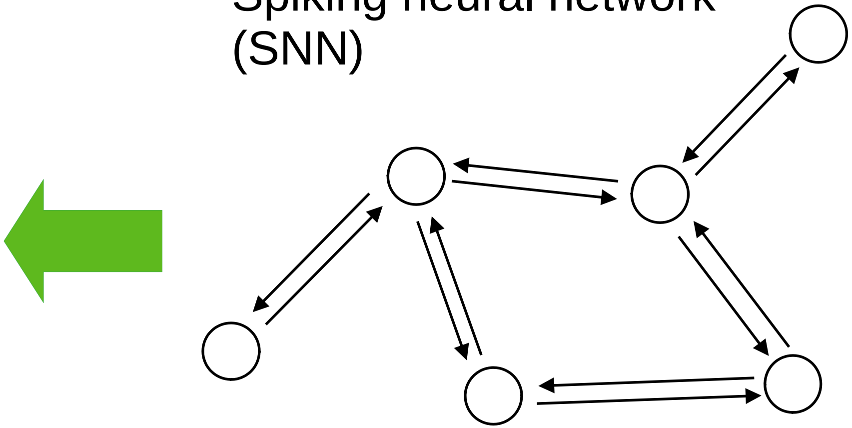

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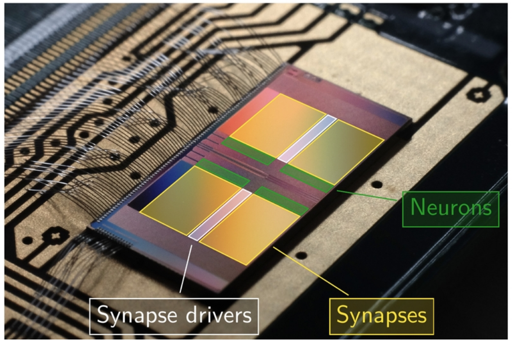

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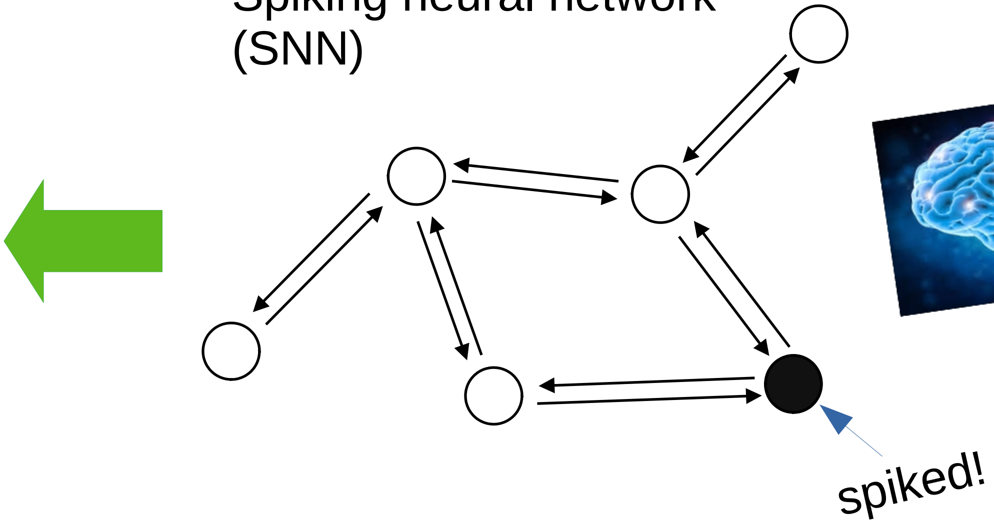

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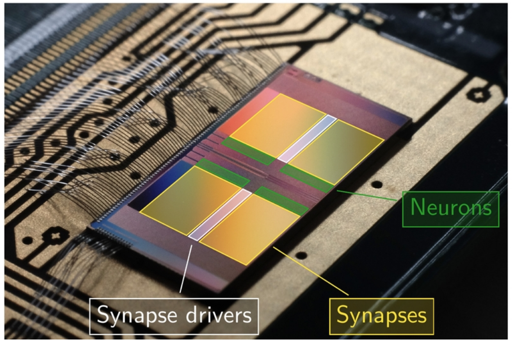

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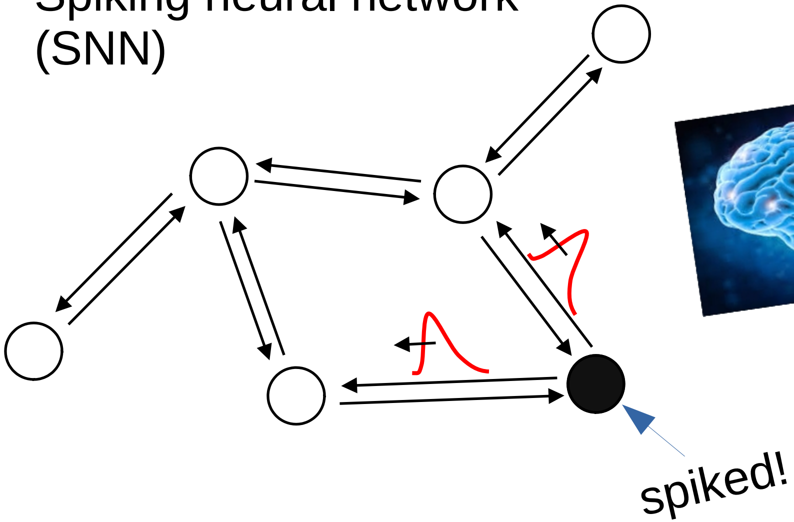

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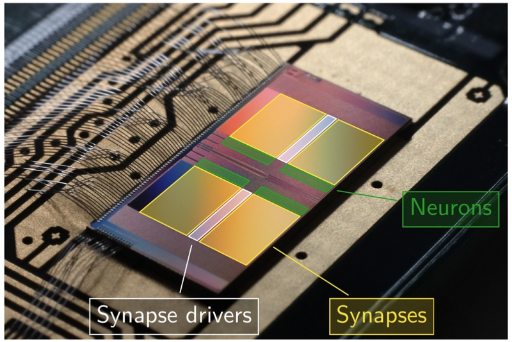

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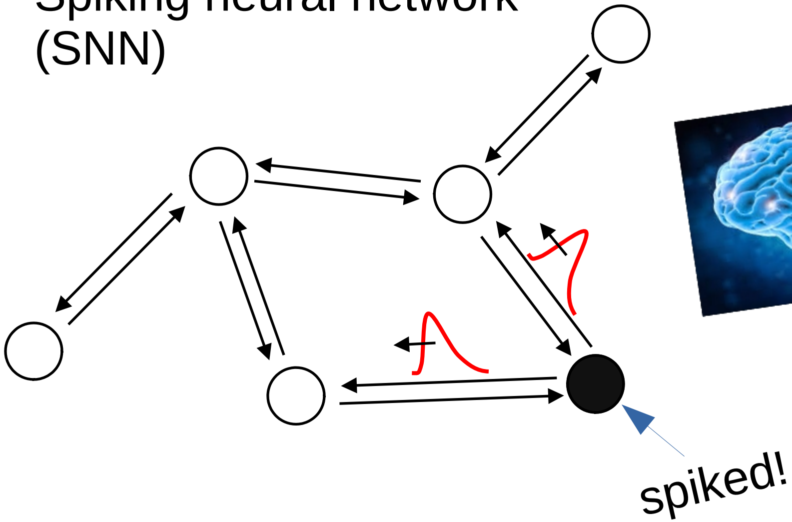

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- inherent parallelism enables fast sampling independent of network size

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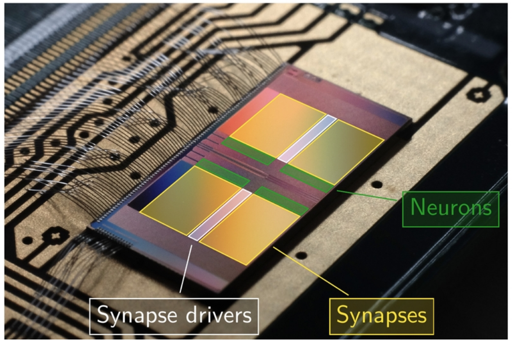

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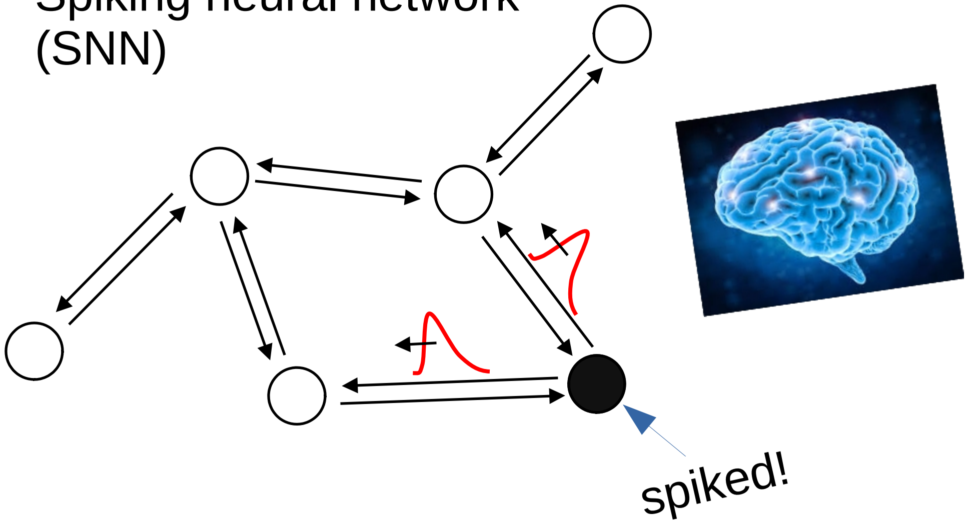

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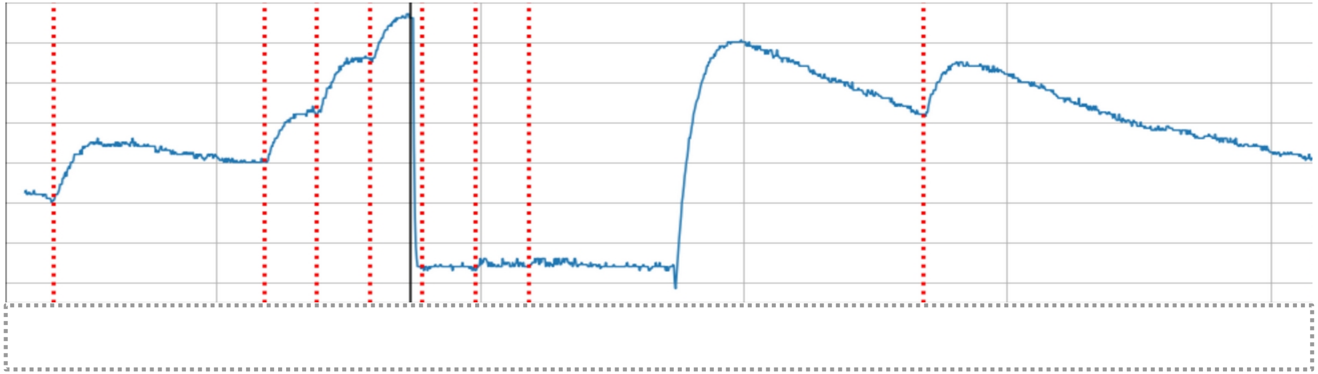
- recently *Czischek et al., arXiv:2008.01039 (2020)*, show high-fidelity encoding of Bell pairs

Neural sampling with dynamic spiking neuron models

dynamics of single neuron's membrane potential u

$u(t)$



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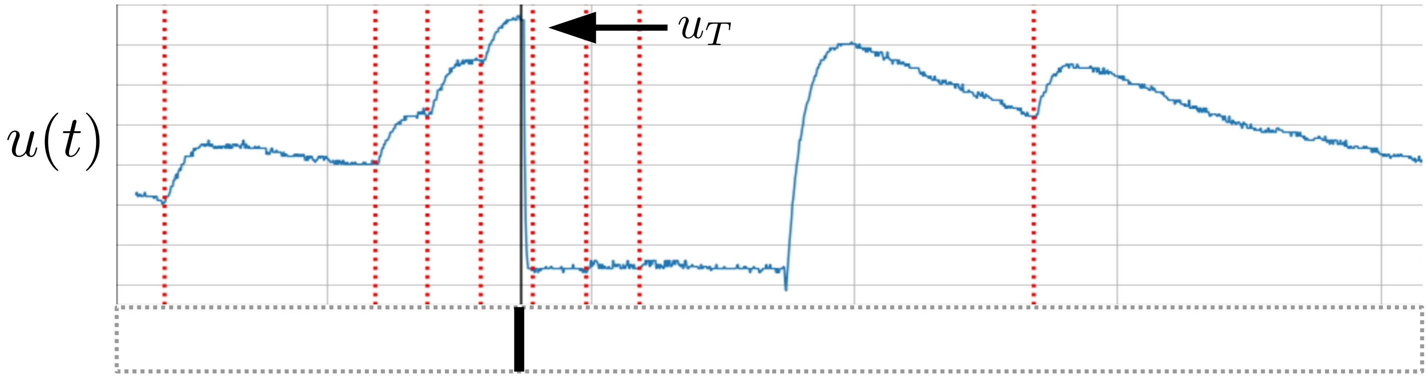

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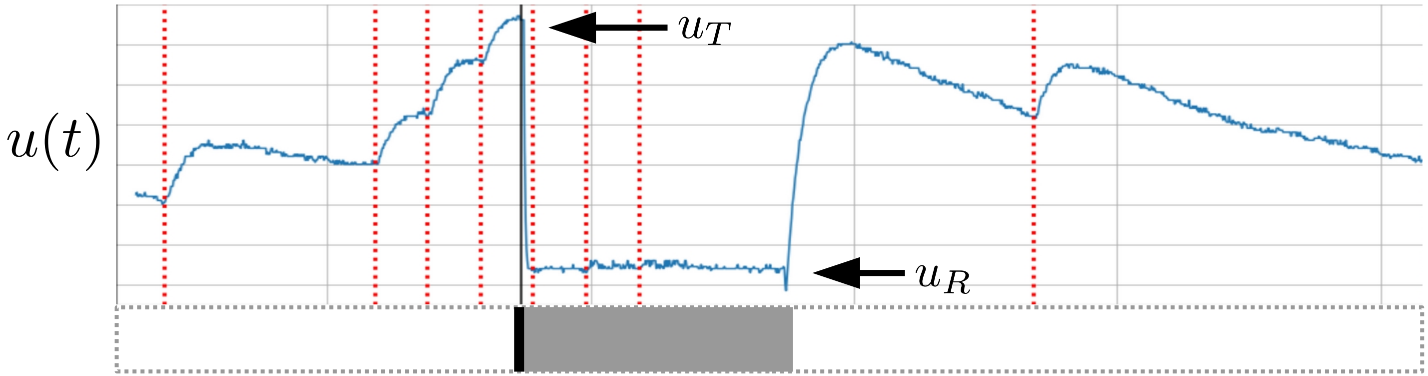

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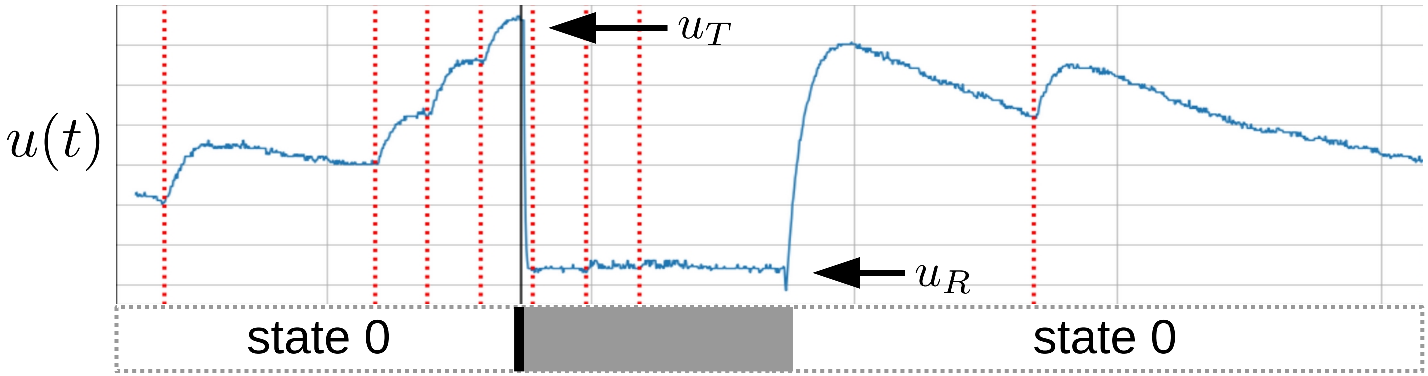

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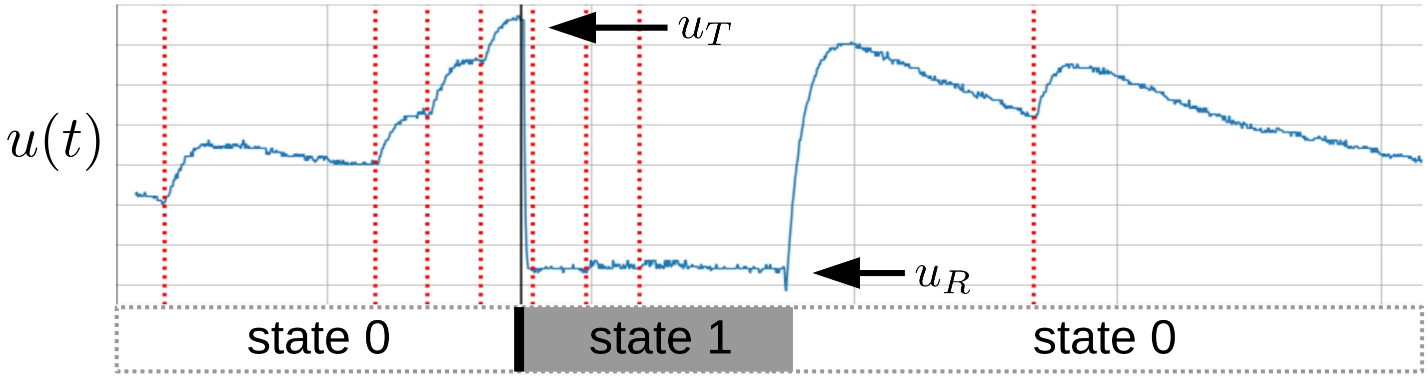

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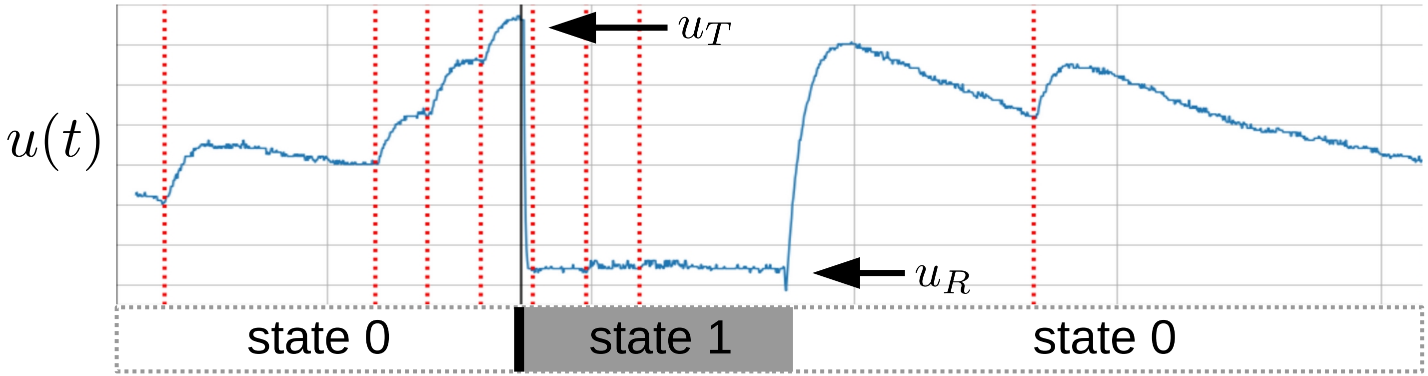

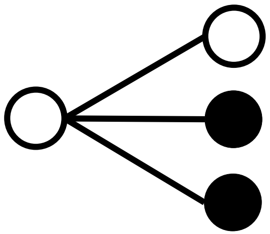

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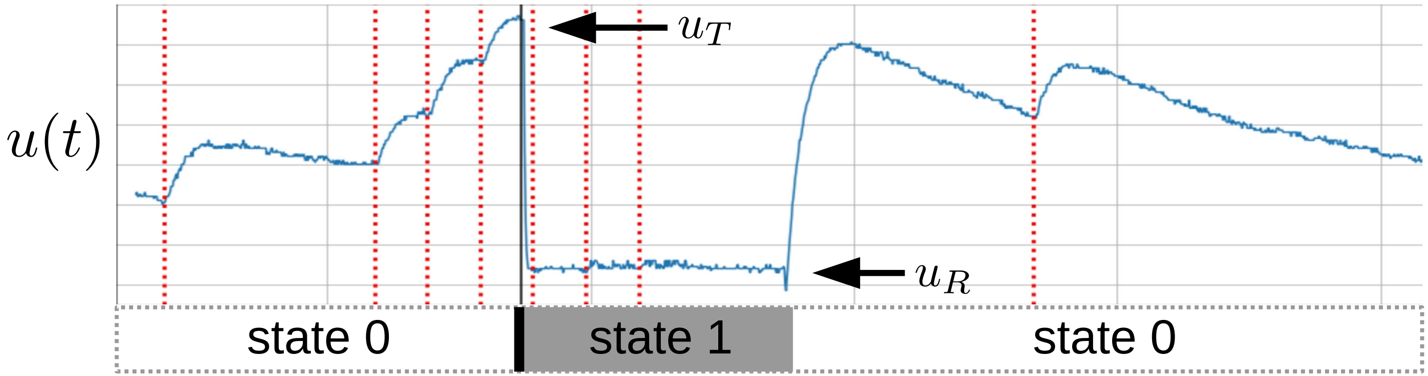

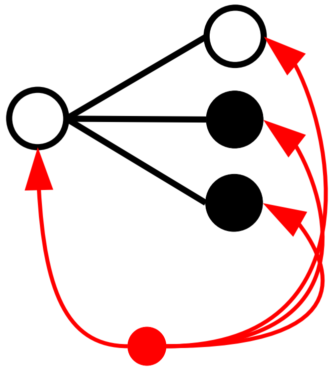

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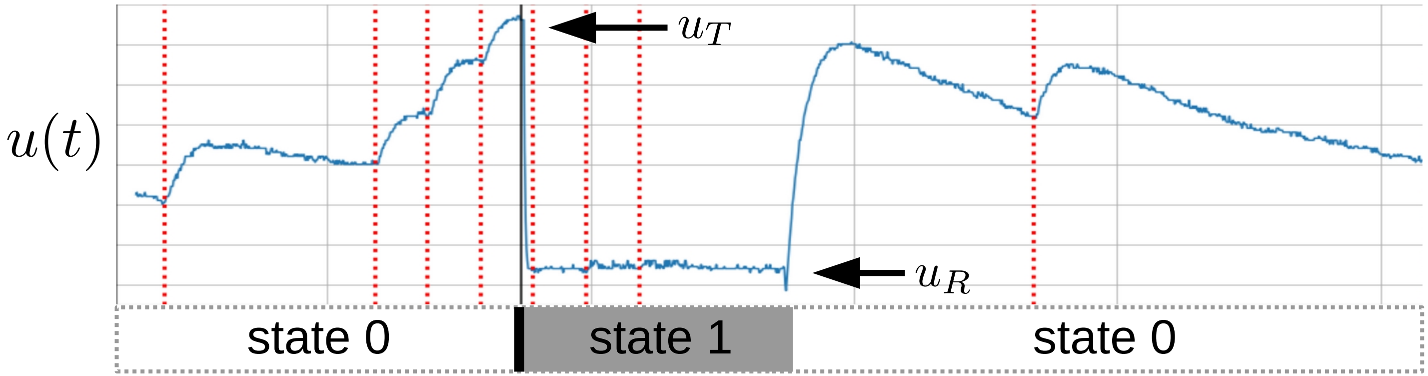

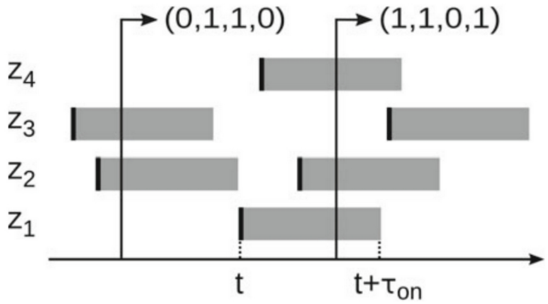

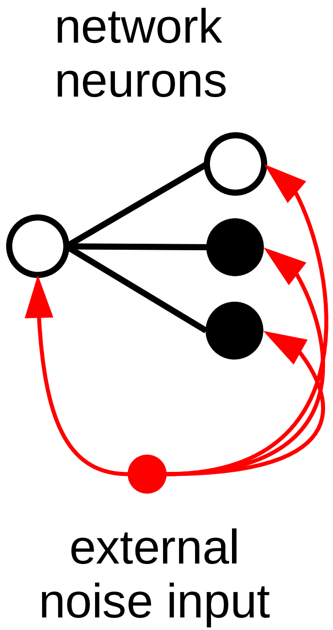

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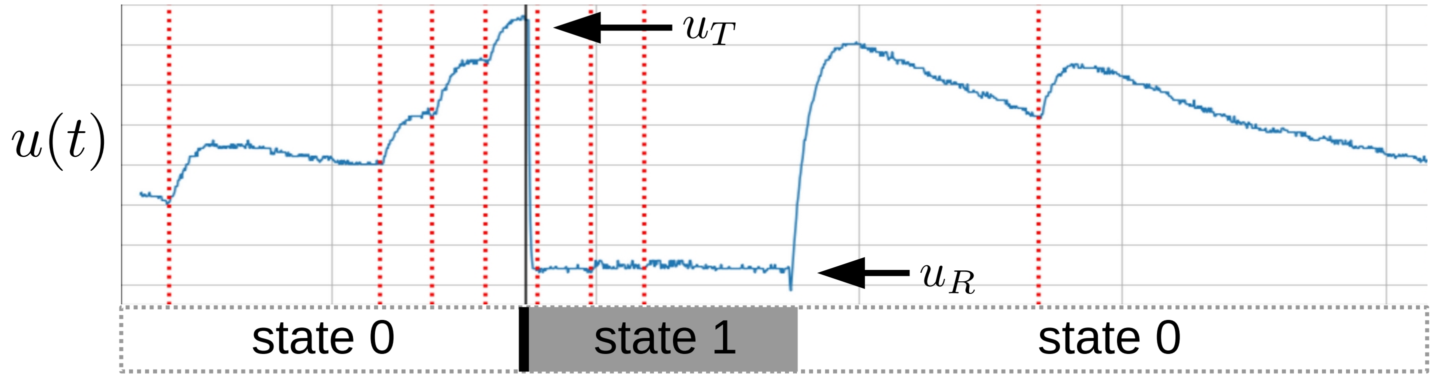
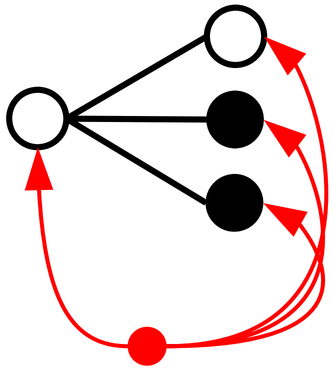
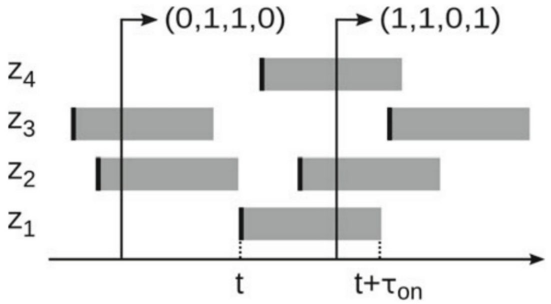


network neurons



external noise input

- sparse spike data represent samples



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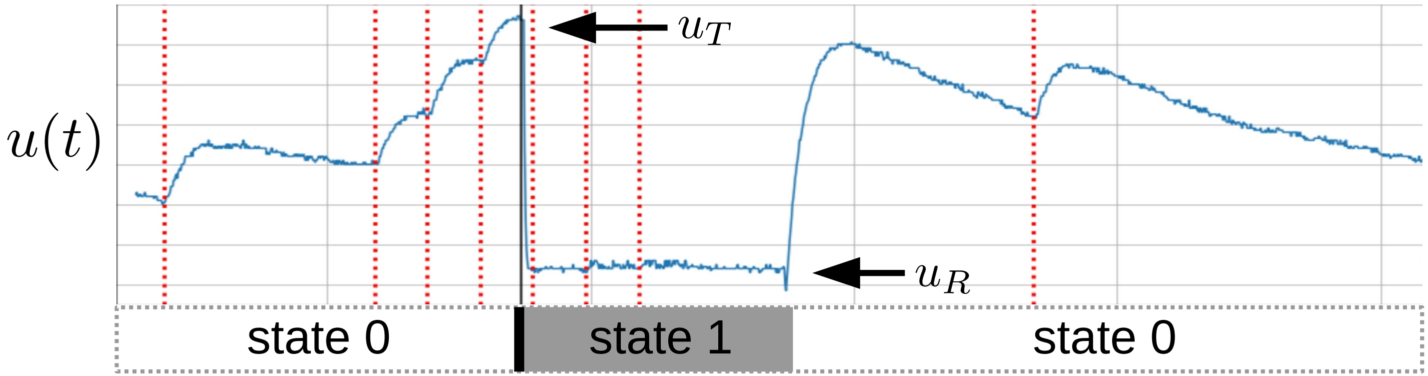

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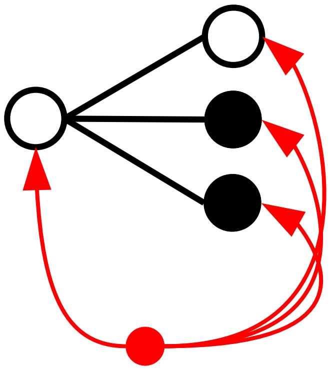

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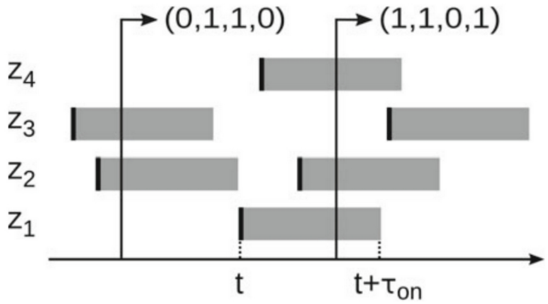

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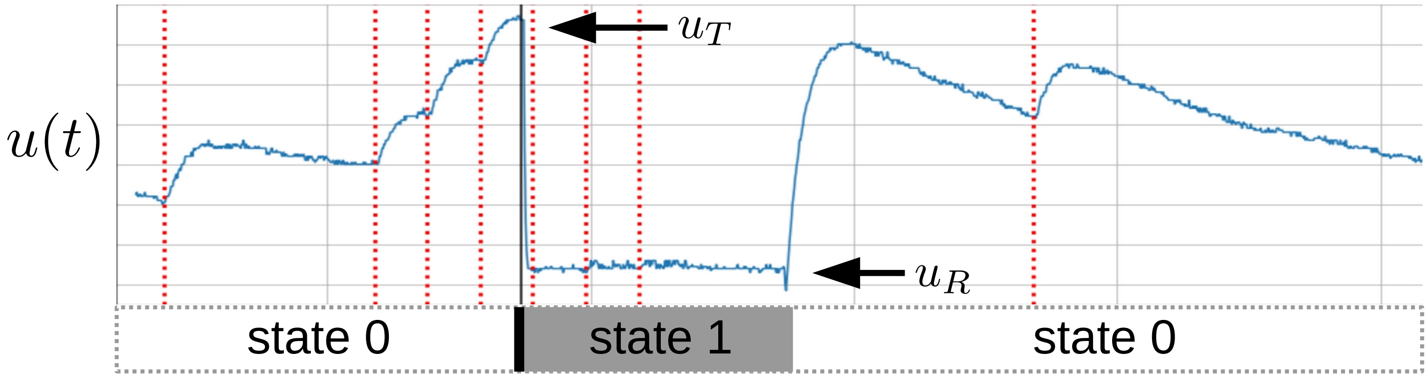

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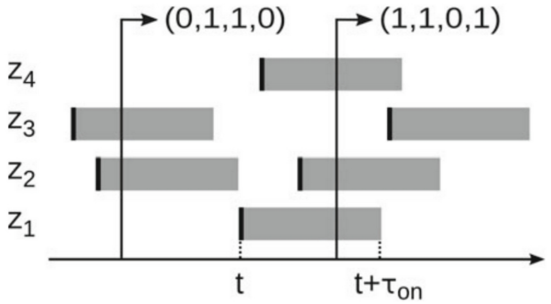

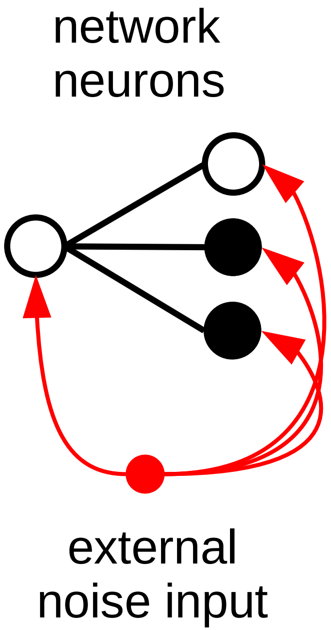

Encoding the quantum states

$$H_{\text{TFIM}} = -J \sum_{\langle i,j \rangle}^{N} \sigma_z^i \sigma_z^j - h \sum_{i}^{N} \sigma_x^i$$

1D, periodic boundary conditions

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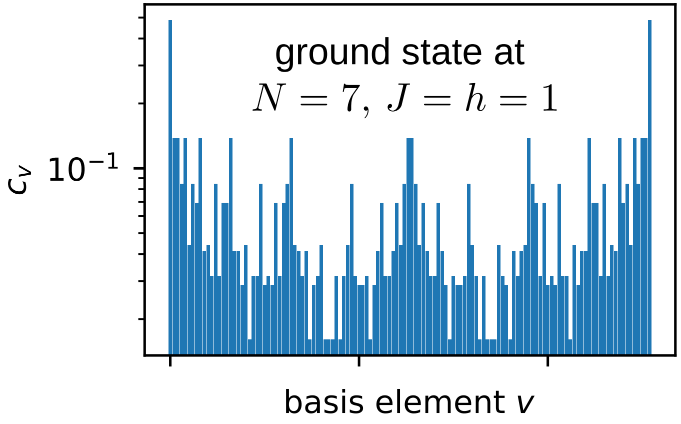

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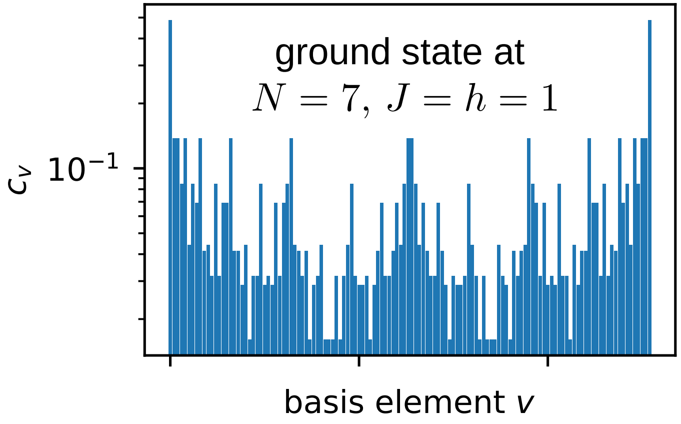

Learning algorithm

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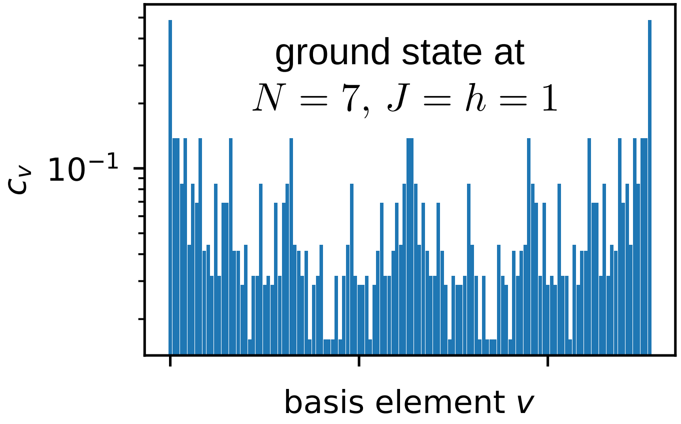

Learning algorithm

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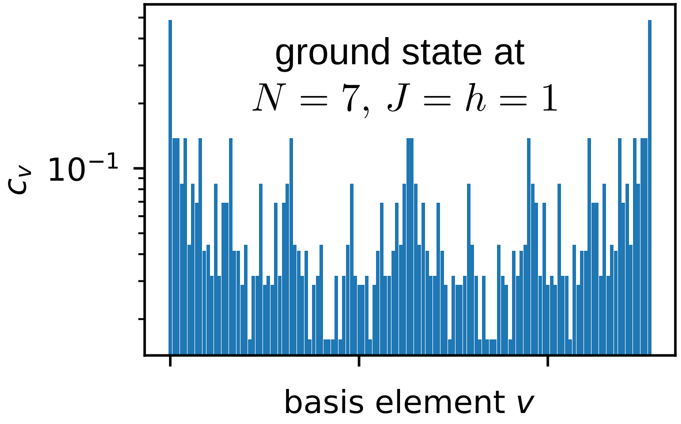

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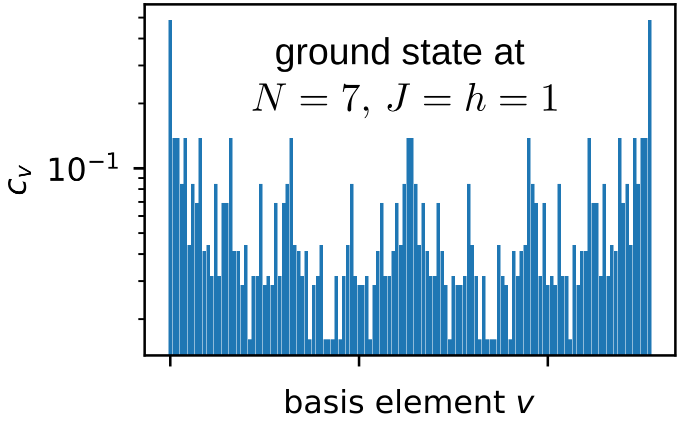

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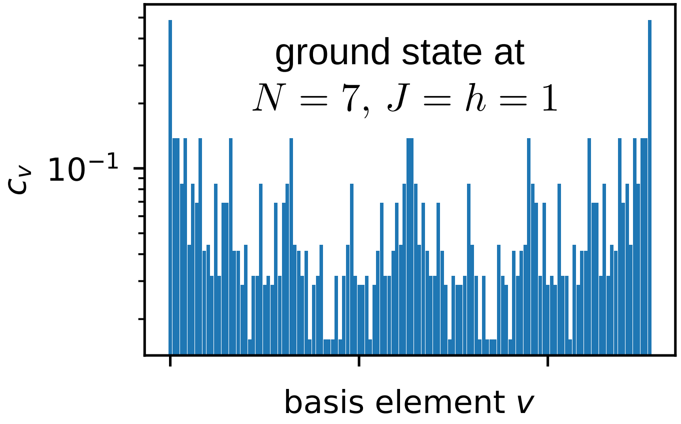

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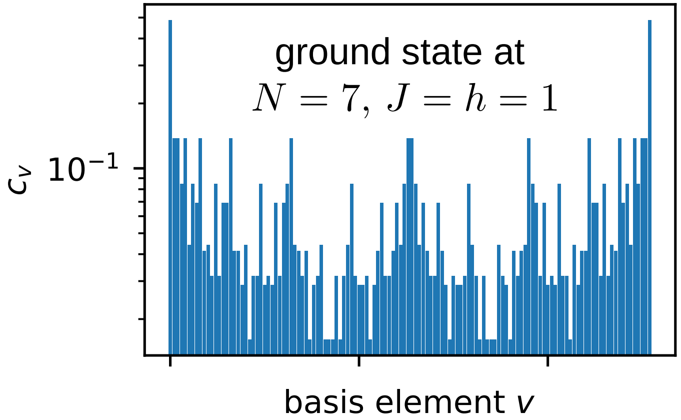

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(4) Update $\theta \leftarrow \theta - \eta \Delta\theta$

Host

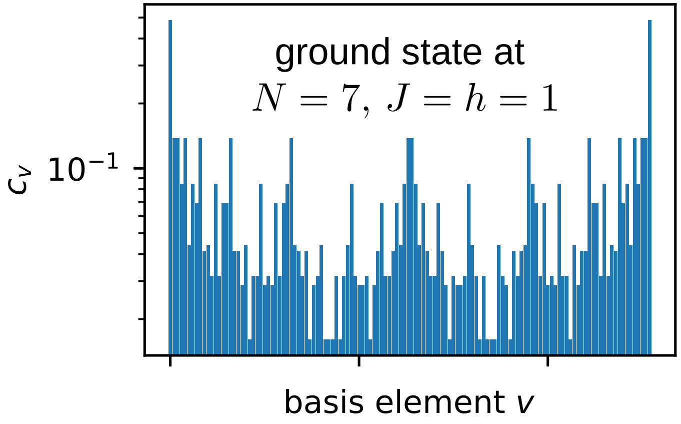
Encoding the quantum states

$$H_{\text{TFIM}} = -J \sum_{\langle i,j \rangle}^{N} \sigma_z^i \sigma_z^j - h \sum_{i}^{N} \sigma_x^i$$

1D, periodic boundary conditions

stoquastic H $\rightarrow$ $c_v \in \mathbb{R}_{\geq 0}$

ansatz: $c_v = \sqrt{p_\theta(v)}$



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(2) Sample $v, h \sim p_\theta$
BSS2

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variational energy

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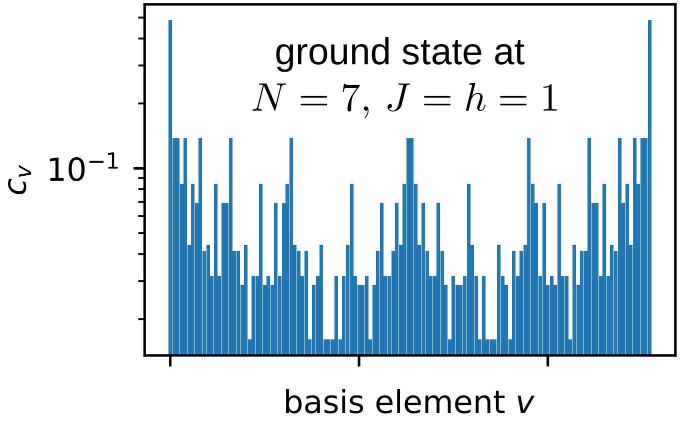

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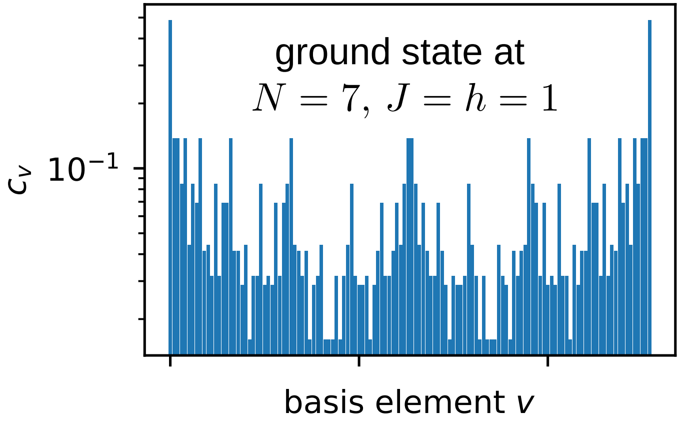

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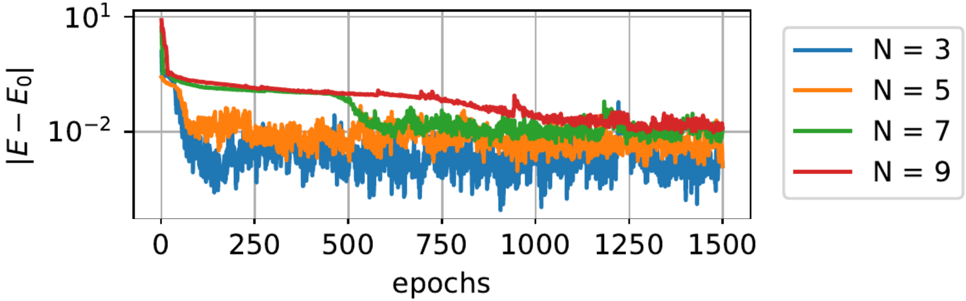
Results



all figures at critical point $J = h = 1$

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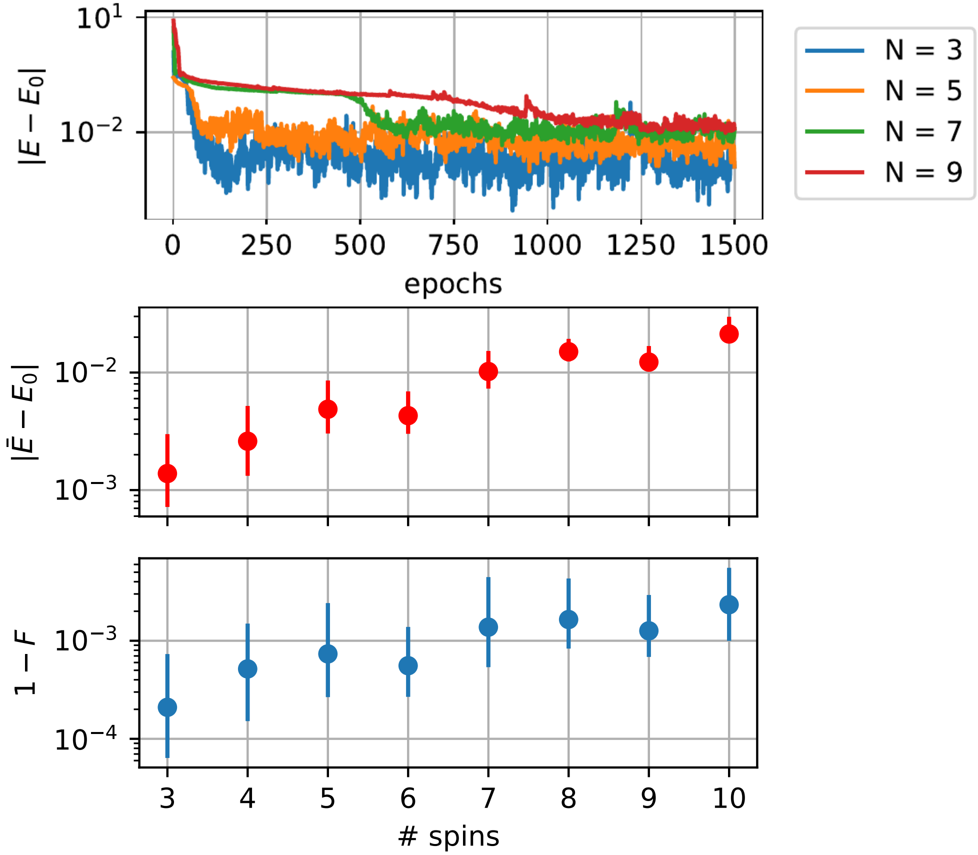

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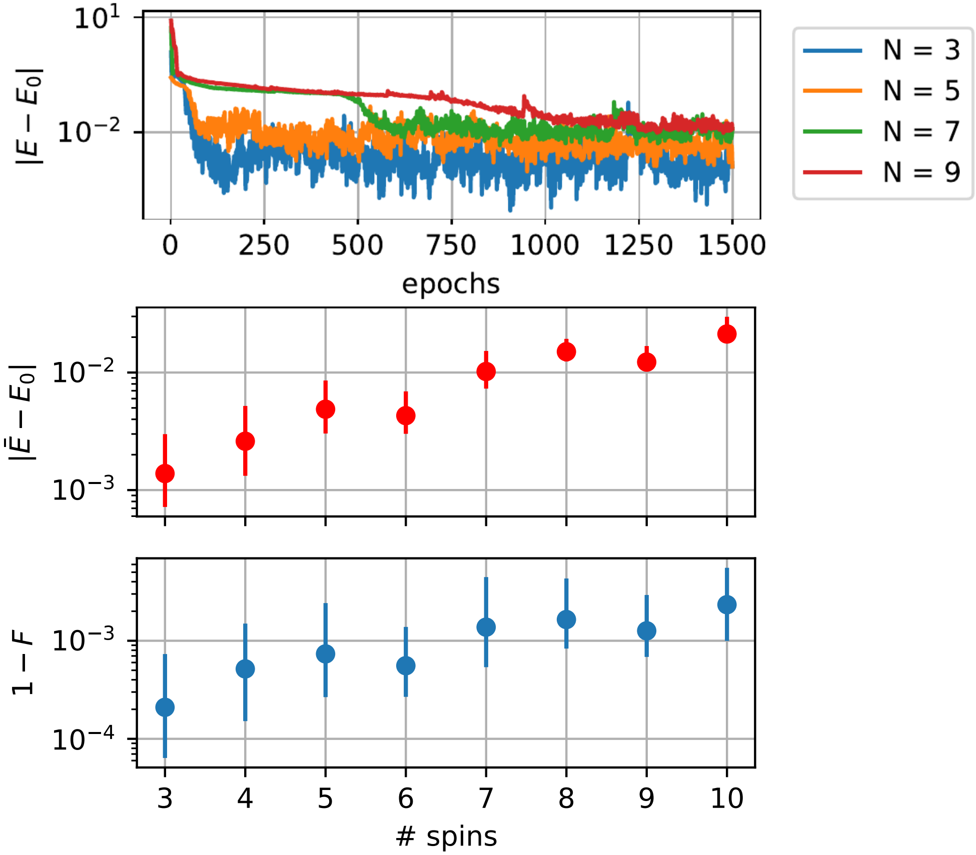

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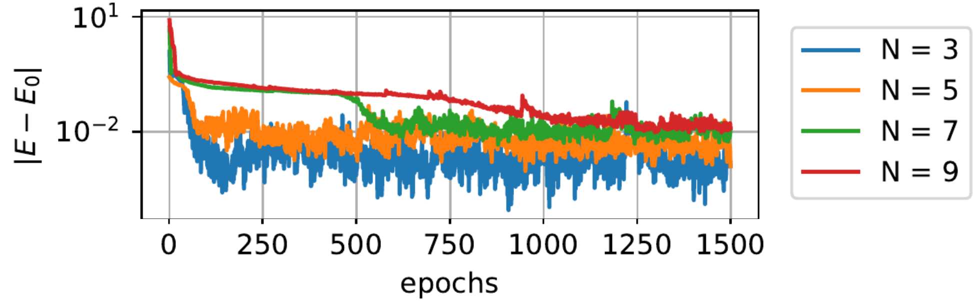

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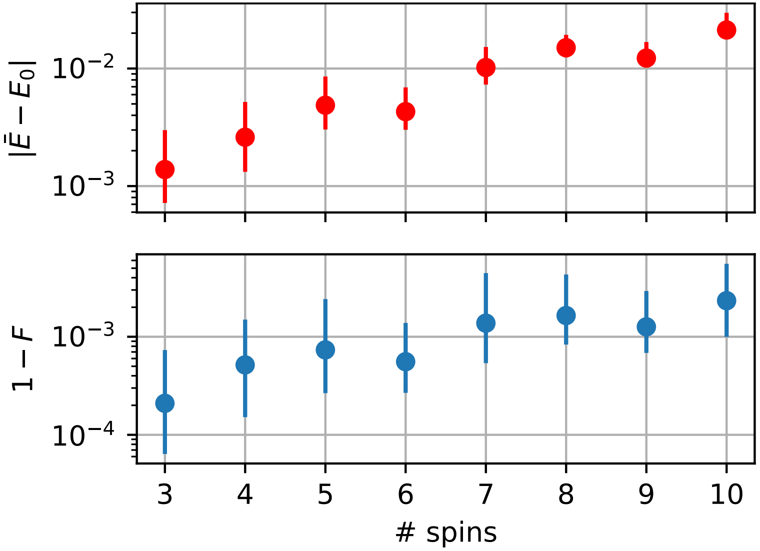

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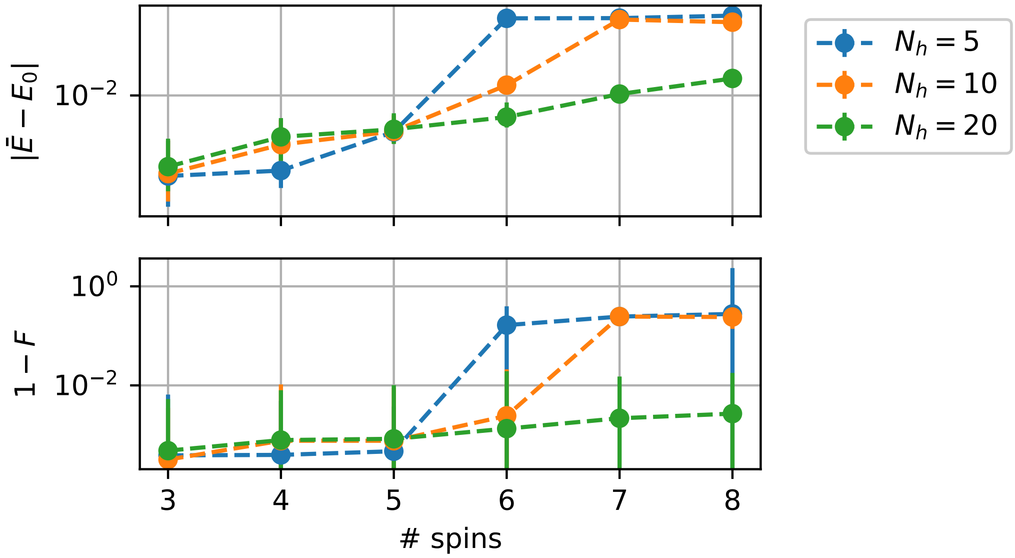

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Summary

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Thank you!

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