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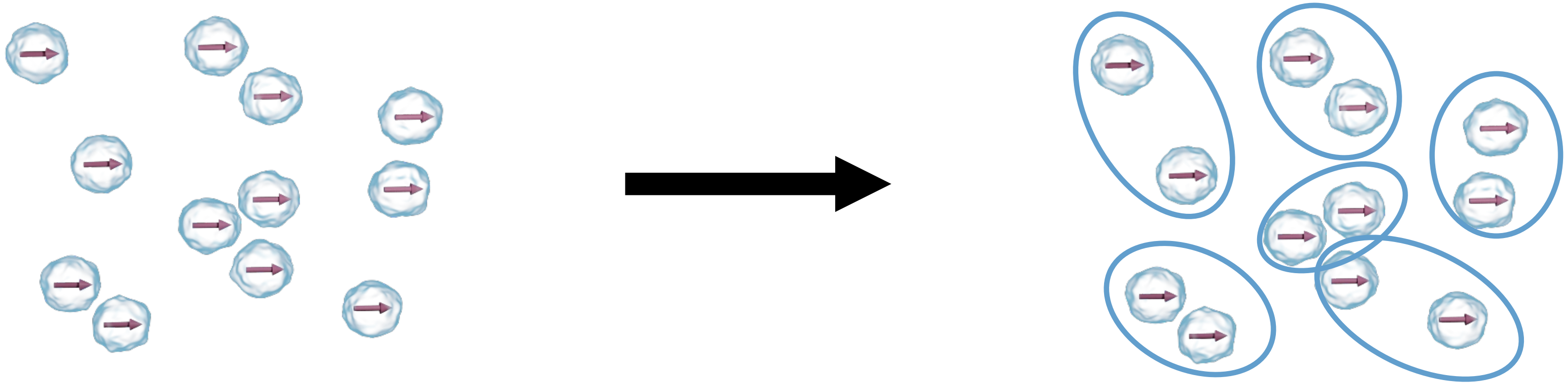

Emergent integrability in Heisenberg spin models with disordered couplings

Adrian Braemer – Göttingen 27.09.2023

The Teams

Theory: Gärttner Group

Now moved to Jena!

Experiment: Weidemüller QD Group

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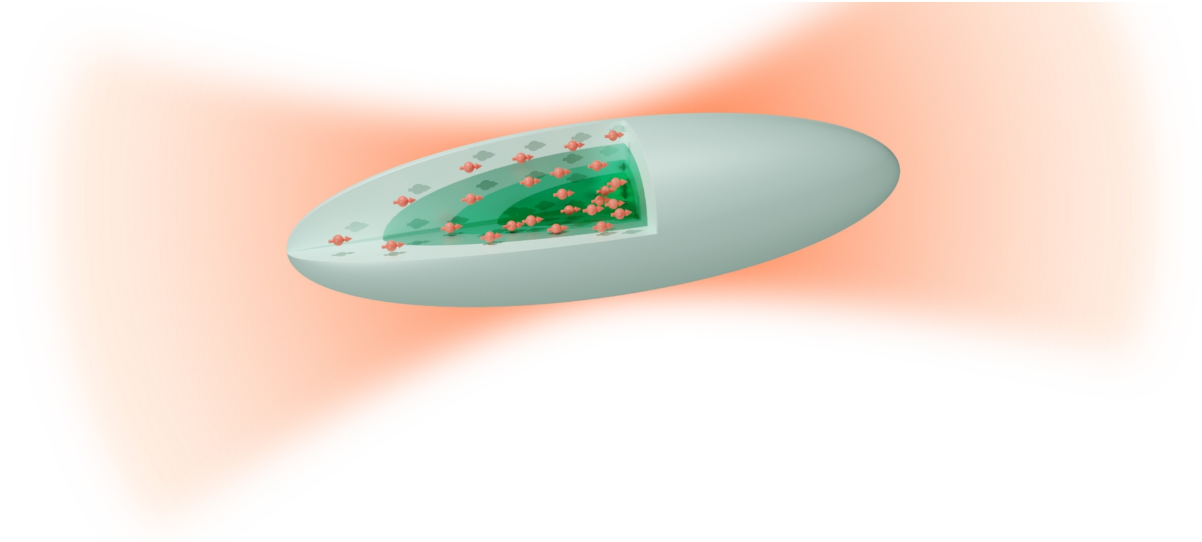
Outline

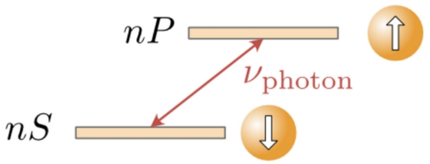
- Introduction of model
- Theory: Localization of pairs
- Experiment: Universal relaxation dynamics
- Theory: Hierarchical pair model (outlook)

The Experiment

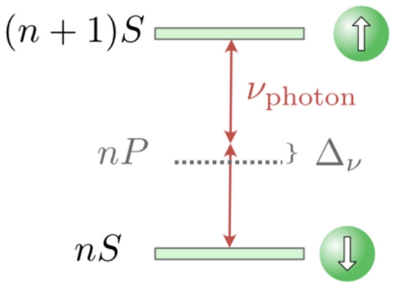
- Cold gas of Rb-87
- Rydberg states encode spins
- Fixed random positions
- Global control



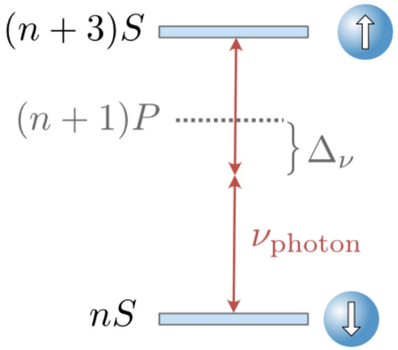
XX



XXZ



Ising

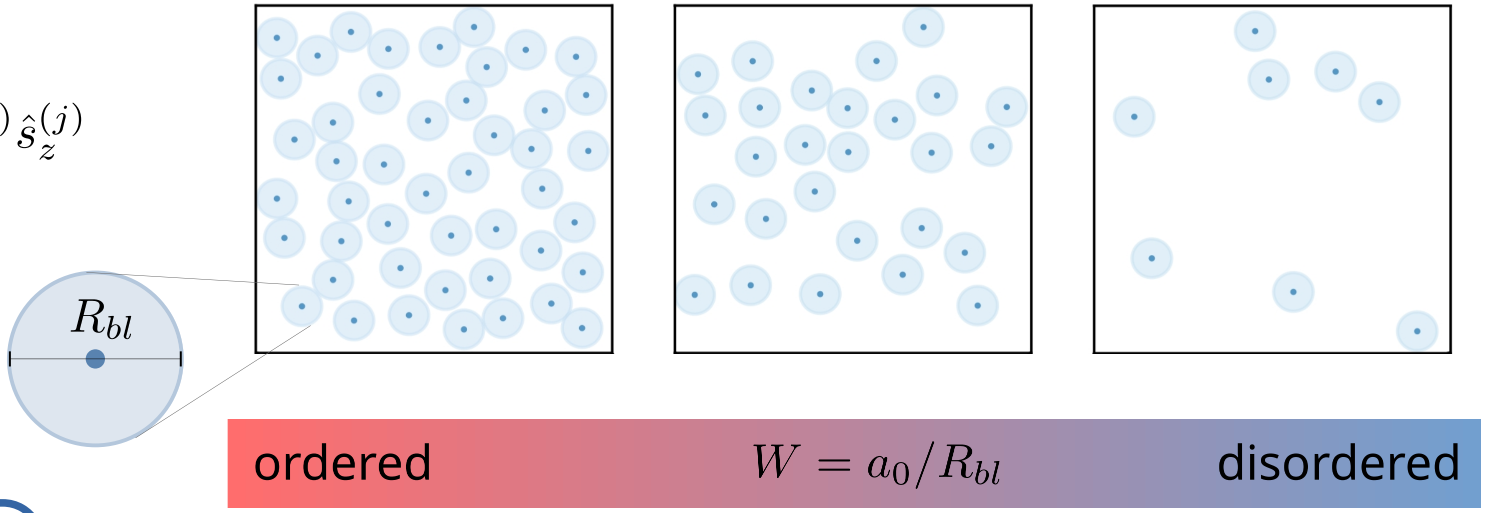


The Experiment (theorist's perspective)

Bond disordered Heisenberg model:

$$\hat{H}_{XXZ} = \sum_{i,j} J_{ij}^{\perp} \left(\hat{s}_+^{(i)} \hat{s}_-^{(j)} + \text{h.c.} \right) + J_{ij}^{\parallel} \hat{s}_z^{(i)} \hat{s}_z^{(j)}$$

$$J_{ij}^{\parallel/\perp} \propto \frac{1}{|r_i - r_j|^{\alpha}}$$



Tunable disorder → Localization?
Simulation: 1d, α=6, PBC, N≤16

Related model on lattice:
Mohdeb *et al.* Arxiv:2303.02415

Localization

$$r_i = \frac{\min(E_{i+1} - E_i, E_i - E_{i-1})}{\max(E_{i+1} - E_i, E_i - E_{i-1})}$$

Braemer *et al.*, PRB **106**,134212 (2022)

Thermal – Level repulsion

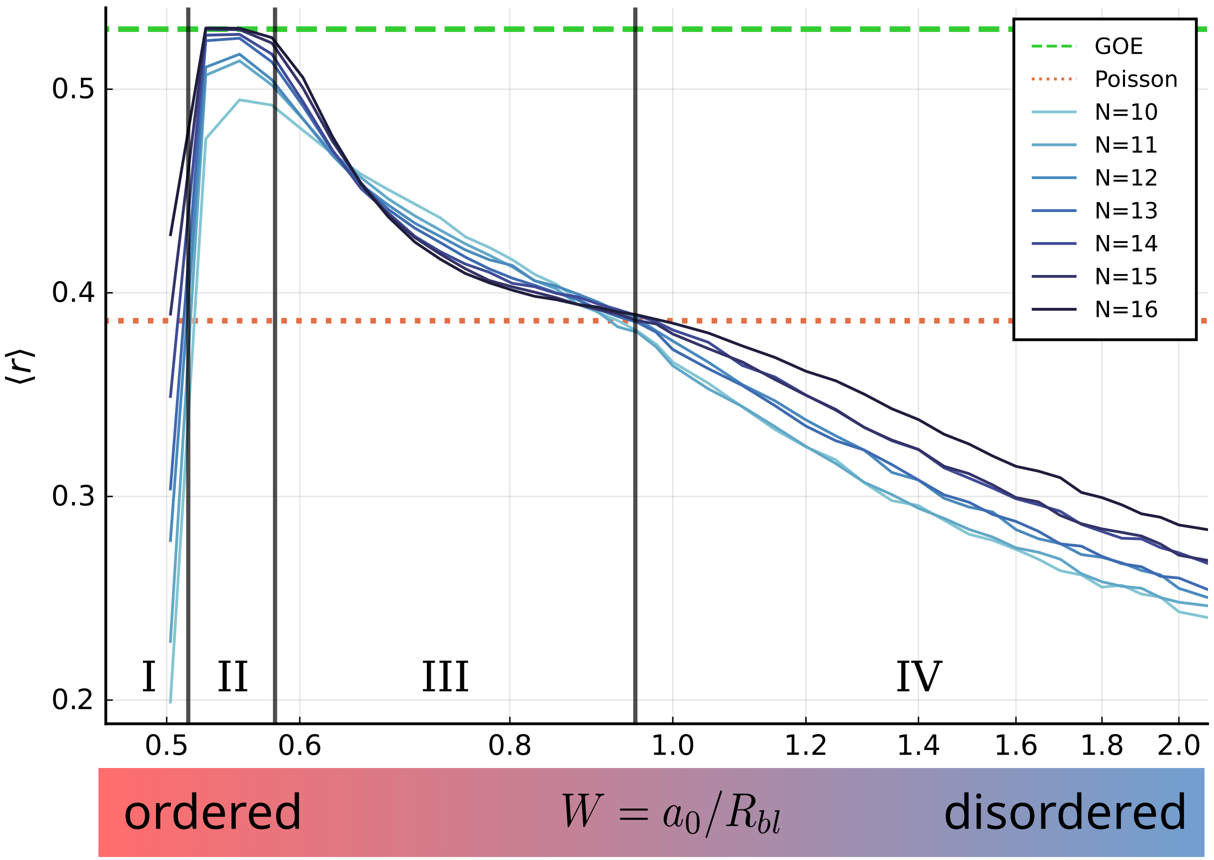
$\langle r \rangle \approx 0.52$

MBL – Poisson

$\langle r \rangle \approx 0.39$

Level attraction?

$\langle r \rangle < 0.39$

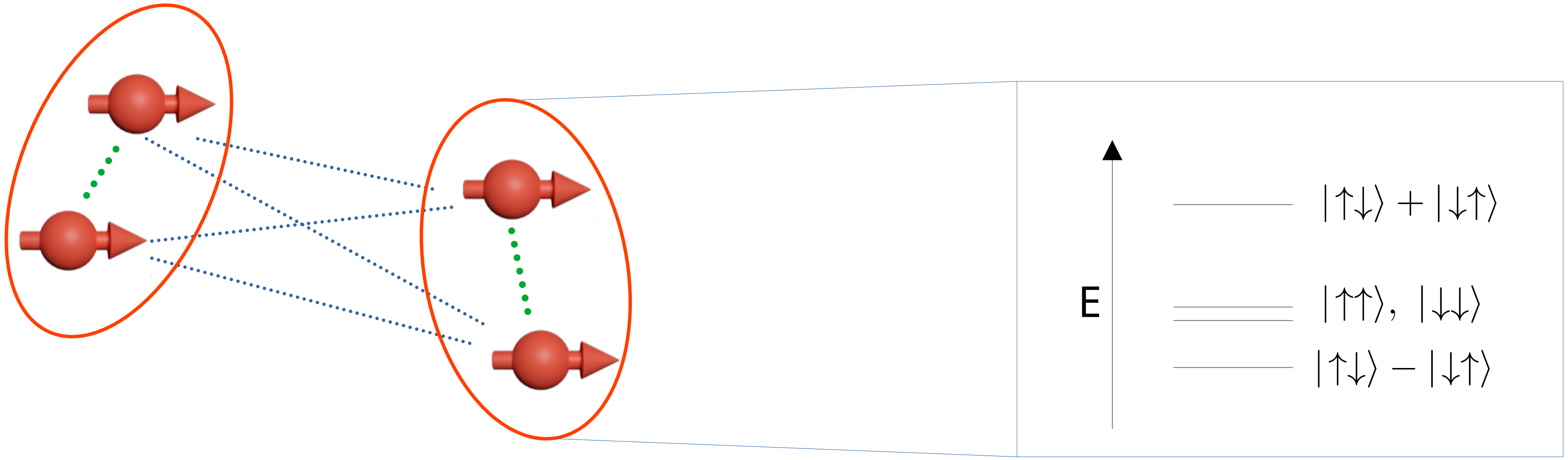


$d = 1, \alpha = 6$

LIOMs: Pair model

Braemer *et al.*,
PRB **106**,134212
(2022)

$$\hat{H} = \sum_{1\le i,j\le 4} \hat{H}^{(i)(j)}_{\text{pair}} \longrightarrow \hat{H} \approx \underbrace{\boxed{J_{12}\hat{H}^{(1)(2)}_{\text{pair}} + J_{34}\hat{H}^{(3)(4)}_{\text{pair}}}}_{\text{decoupled pairs}} + \underbrace{\boxed{\tilde{\Delta}(\hat{s}^{(1)}_z + \hat{s}^{(2)}_z)(\hat{s}^{(3)}_z + \hat{s}^{(4)}_z)}}_{\text{effective Ising term}}$$

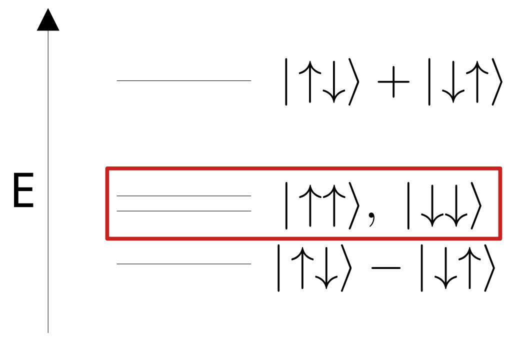


Emergent integrability!

Localization

$$r_i = \frac{\min(E_{i+1} - E_i, E_i - E_{i-1})}{\max(E_{i+1} - E_i, E_i - E_{i-1})}$$

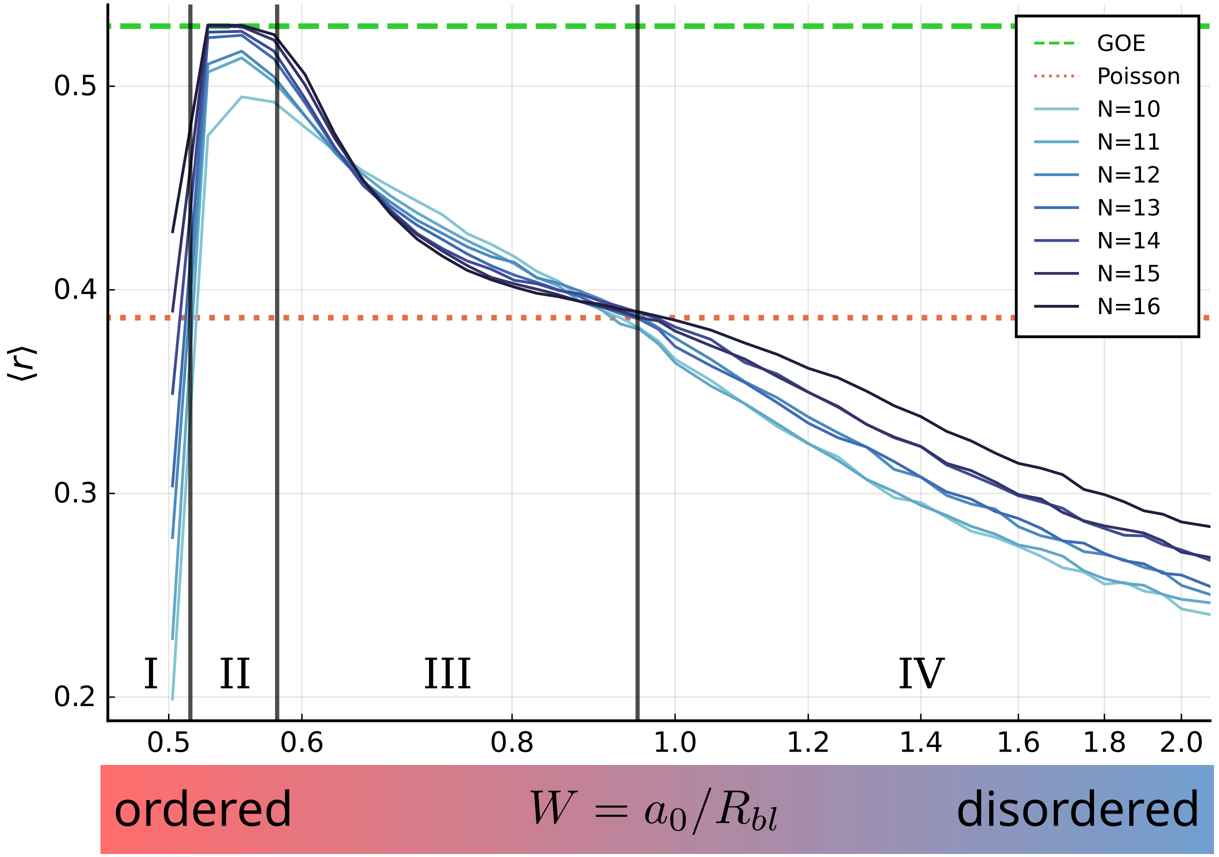
Braemer *et al.*, PRB **106**,134212 (2022)



degenerate

Level attraction

$\langle r \rangle < 0.39$

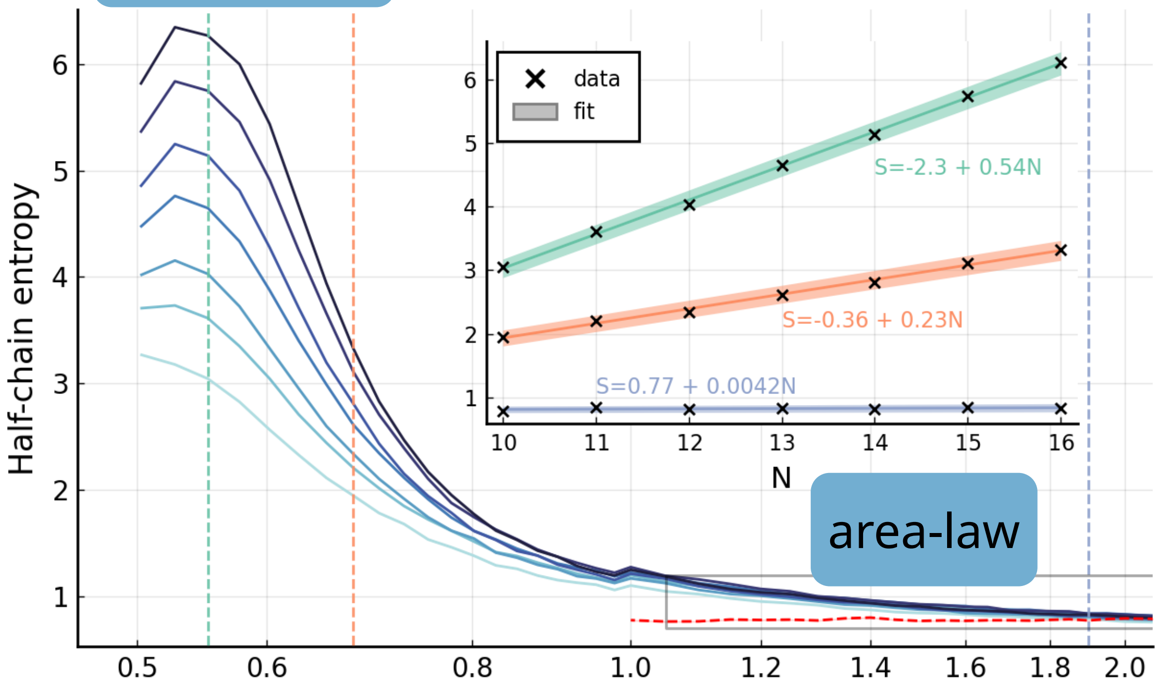


$d = 1, \alpha = 6$

Half-chain Entanglement Entropy

Braemer *et al.*, PRB **106**,134212 (2022)

volume-law

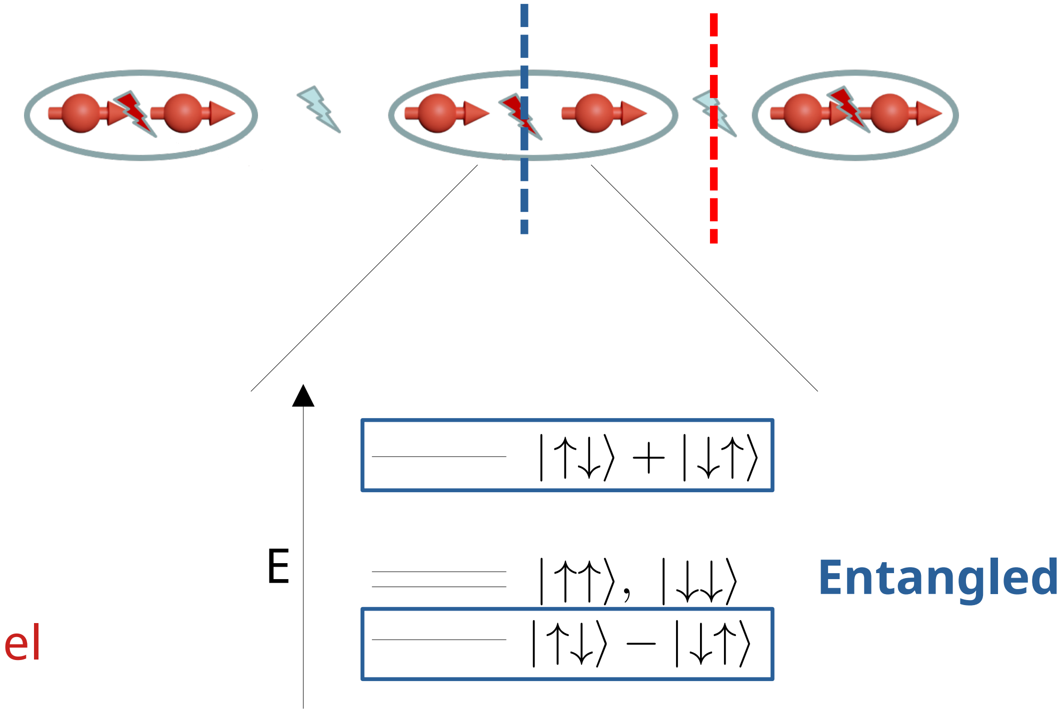


ordered $W = a_0/R_{bl}$ disordered

$d = 1, \alpha = 6$

No entanglement

pair model



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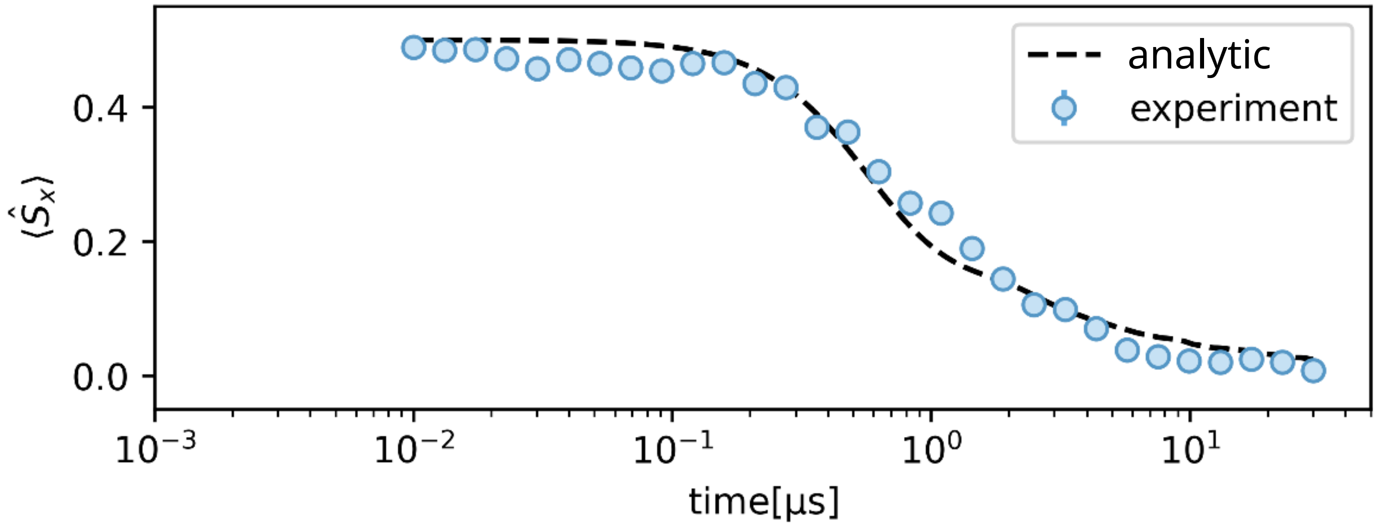
Relaxation dynamics

A. Signoles, T. Franz *et al.*, PRX **11**, 011011 (2021)
P. Schultzen *et al.*, PRB **105**, L020201 (2022)
P. Schultzen *et al.*, PRB **105**, L100201 (2022)

Excitation	Rotation	Evolution	Readout



Ising model:

$$\hat{H}_{Ising} = \sum_{i,j} J_{ij}^{\parallel} \hat{s}_z^{(i)} \hat{s}_z^{(j)}$$

Emch (1966), Radin (1970):

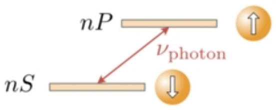
$$\langle S_x \rangle = \frac{1}{2N} \sum_i \prod_j \cos(J_{ij} t)$$

Stretched exponential

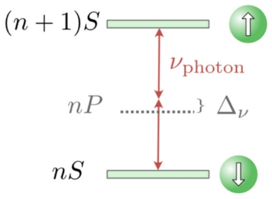
$$\langle S_x(t) \rangle \propto \exp\left[-\left(\frac{t}{\tau} \right)^{\beta} \right]$$

Relaxation in different models

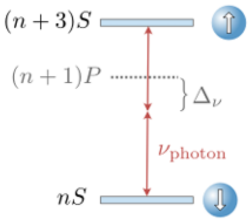
T. Franz, S.Geier *et al.*,
arXiv:2209.08080



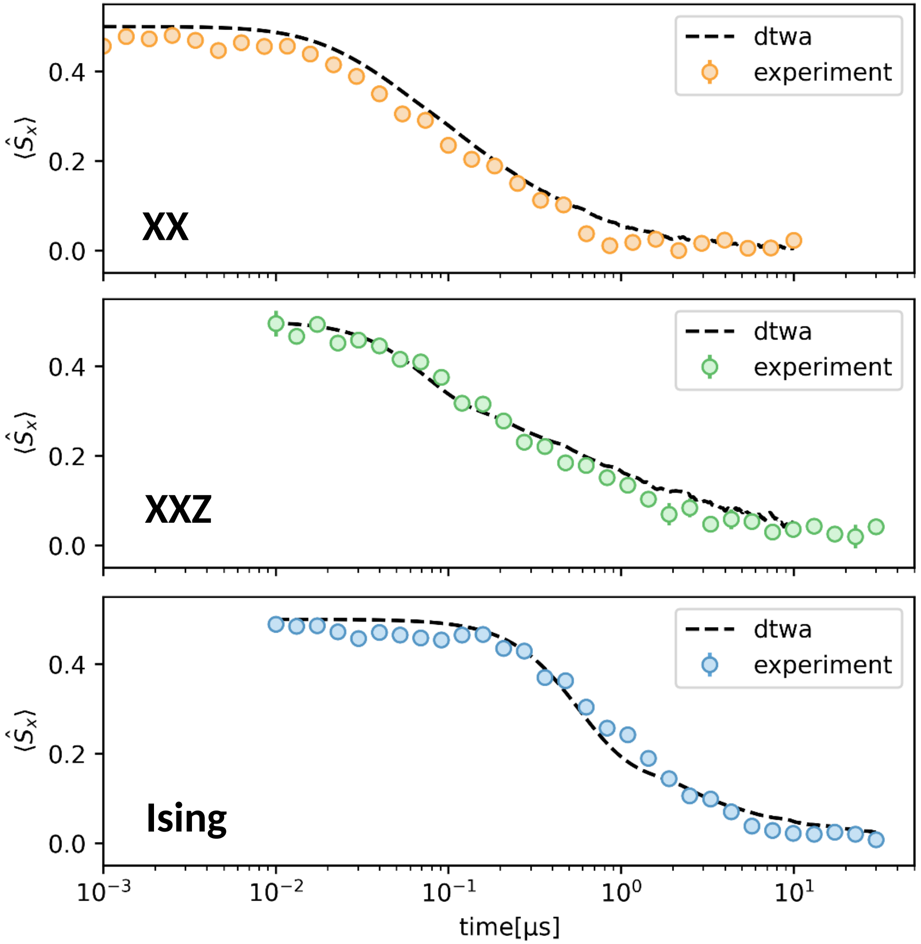
$\boxed{\frac{J_\parallel}{J_\perp} = 0}$ $\quad J \propto \frac{1}{r^3}$



$\boxed{\frac{J_\parallel}{J_\perp} = -0.7}$ $\quad J \propto \frac{1}{r^6}$



$\boxed{\frac{J_\parallel}{J_\perp} = 400}$ $\quad J \propto \frac{1}{r^6}$

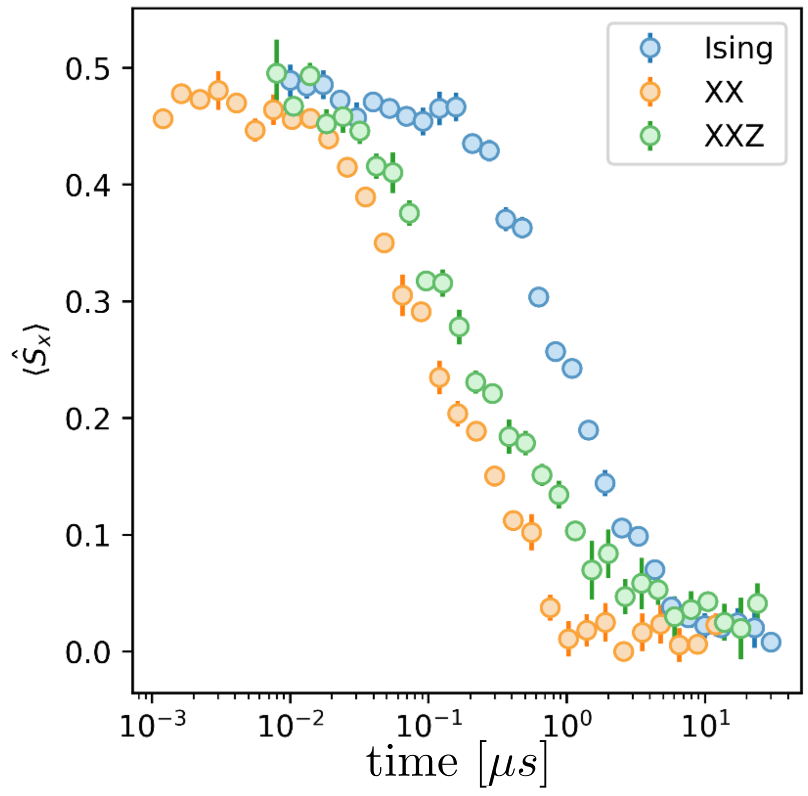


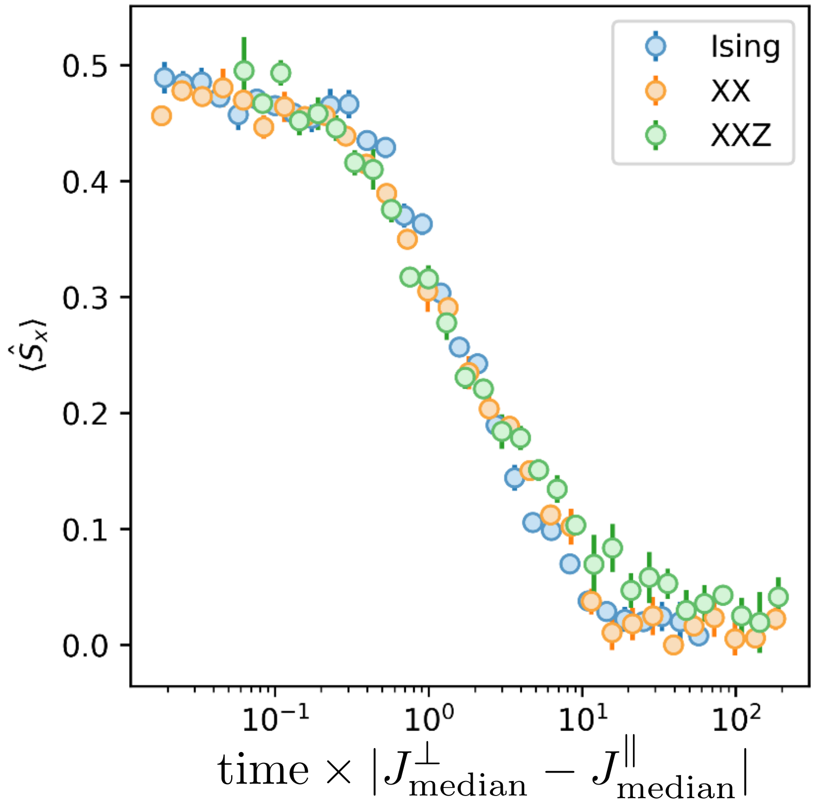
$$\hat{H}_{XXZ} = \sum_{i,j} J_{ij}^{\perp} \left(\hat{s}_+^{(i)} \hat{s}_-^{(j)} + \text{h.c.} \right) + \sum_{i,j} J_{ij}^{\parallel} \hat{s}_z^{(i)} \hat{s}_z^{(j)}$$

$$\hat{H}_{Ising} = \sum_{i,j} J_{ij}^{\parallel} \hat{s}_z^{(i)} \hat{s}_z^{(j)}$$

Universal Relaxation

T. Franz, S.Geier *et al.*, arXiv:2209.08080





$$\hat{H}_{XXZ} = \sum_{i,j} J^{\perp}_{ij} \left(\hat{s}^{(i)}_{+} \hat{s}^{(j)}_{-} + \text{h.c.} \right) + \sum_{i,j} J^{\parallel}_{ij} \hat{s}^{(i)}_{z} \hat{s}^{(j)}_{z}$$

$$\langle S_x(t) \rangle \propto \exp\left[- \left(\frac{t}{\tau} \right)^{\beta} \right]$$

$$\tau \propto |J^{\perp} - J^{\parallel}|$$

$$J^{\perp/\parallel}_{\text{median}} = \text{median}_i \max_j |J^{\perp/\parallel}_{ij}|$$

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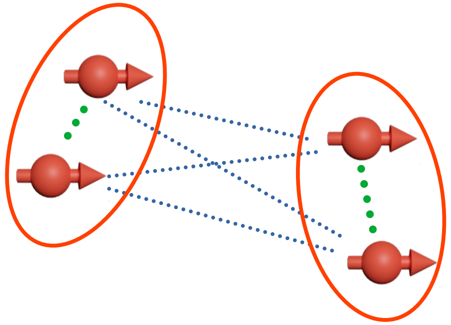
Pair relaxation dynamics

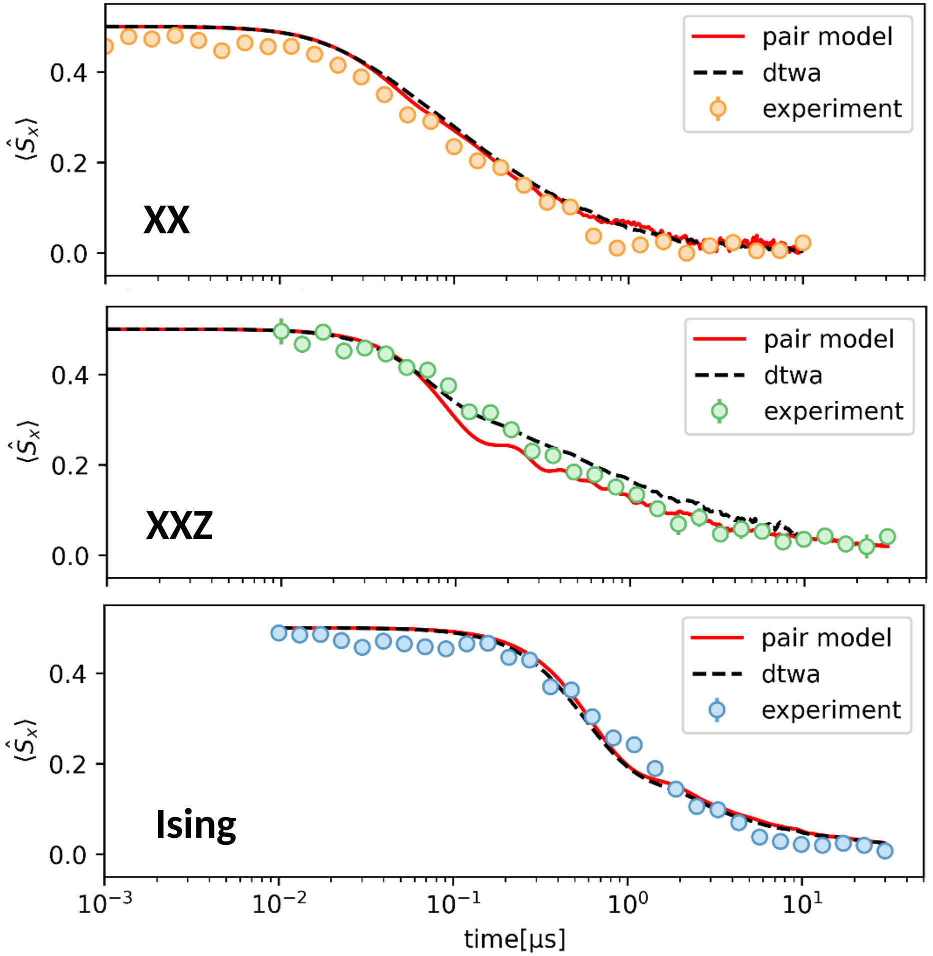
T. Franz, S.Geier *et al.*,
arXiv:2209.08080



$$\langle S_x(t)\rangle = \frac{1}{N}\sum_{<i,j>} \cos\Big(2(J^{\perp}_{ij} - J^{||}_{ij})t\Big) \prod_{<k,l>} \cos^2(J^{ijkl}_{eff}\, t)$$

$$J^{ijkl}_{eff} = \frac{J^{||}_{ik} + J^{||}_{il} + J^{||}_{jk} + J^{||}_{jl}}{2}$$

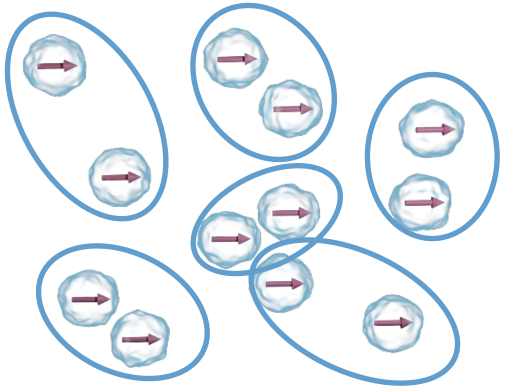
Characteristic timescale $\tau \propto |J^{\perp} - J^{||}|$



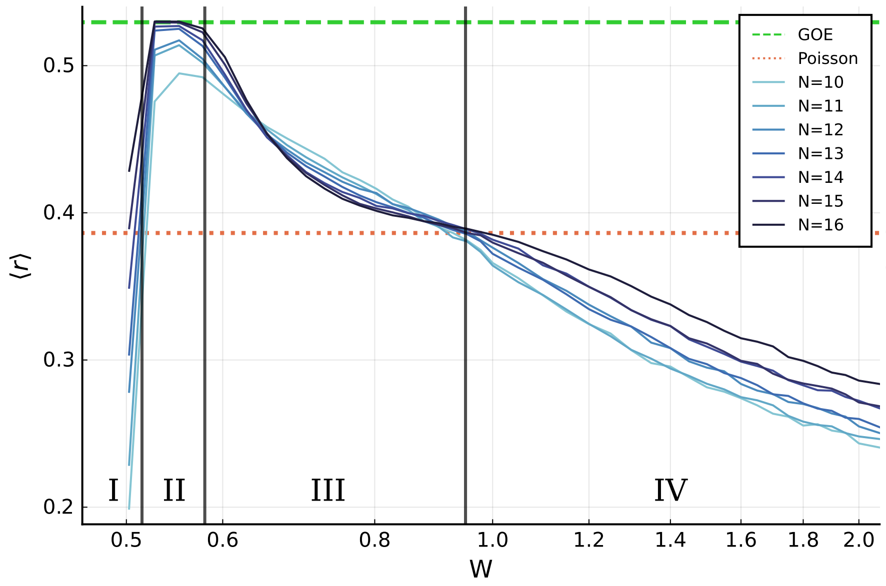
Emergent integrability!

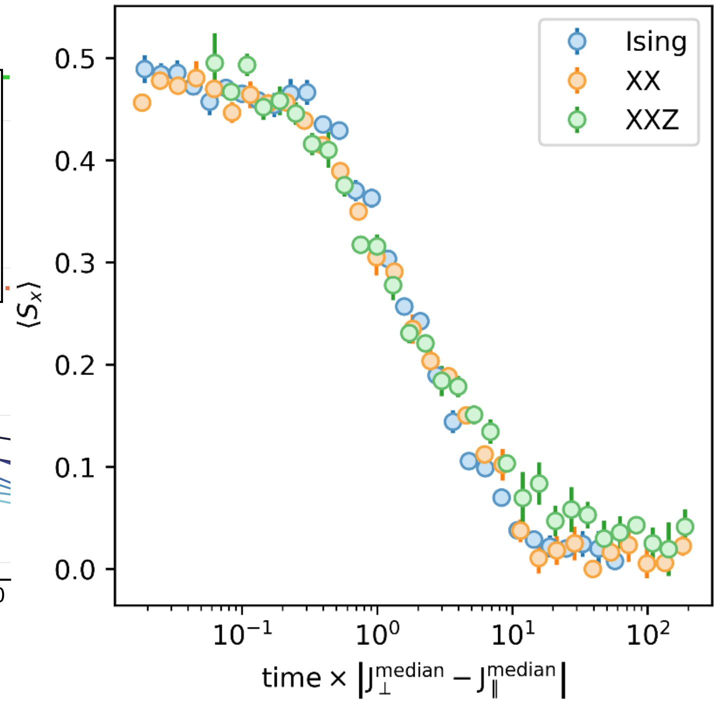
Summary

$$\hat{H}_{XXZ} = \sum_{i,j} J_{ij}^{\perp} \left(\hat{s}_+^{(i)} \hat{s}_-^{(j)} + \text{h.c.} \right) + \sum_{i,j} J_{ij}^{\parallel} \hat{s}_z^{(i)} \hat{s}_z^{(j)}$$



- Disorder leads to emergent integrability
- Pair desciption
- MBL in numerics
- Universal dynamics in experiment



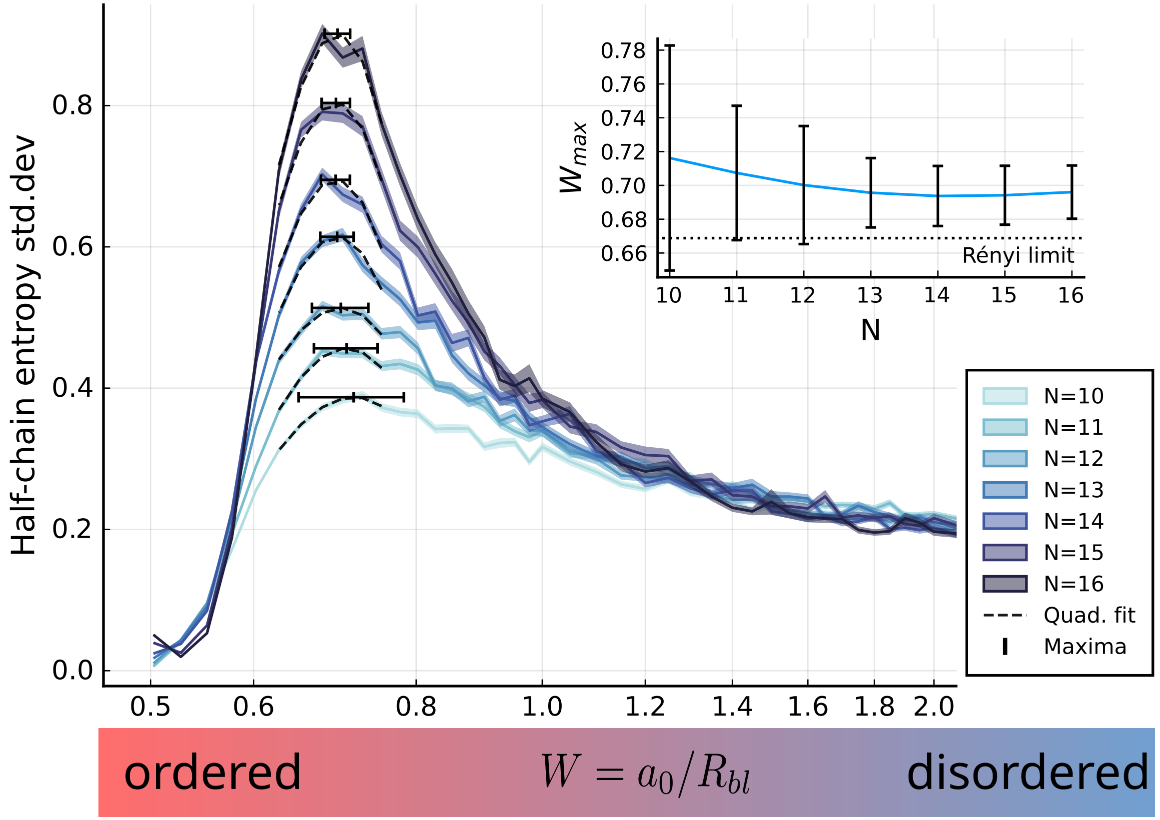


Outlook: Hierarchy of pairs?

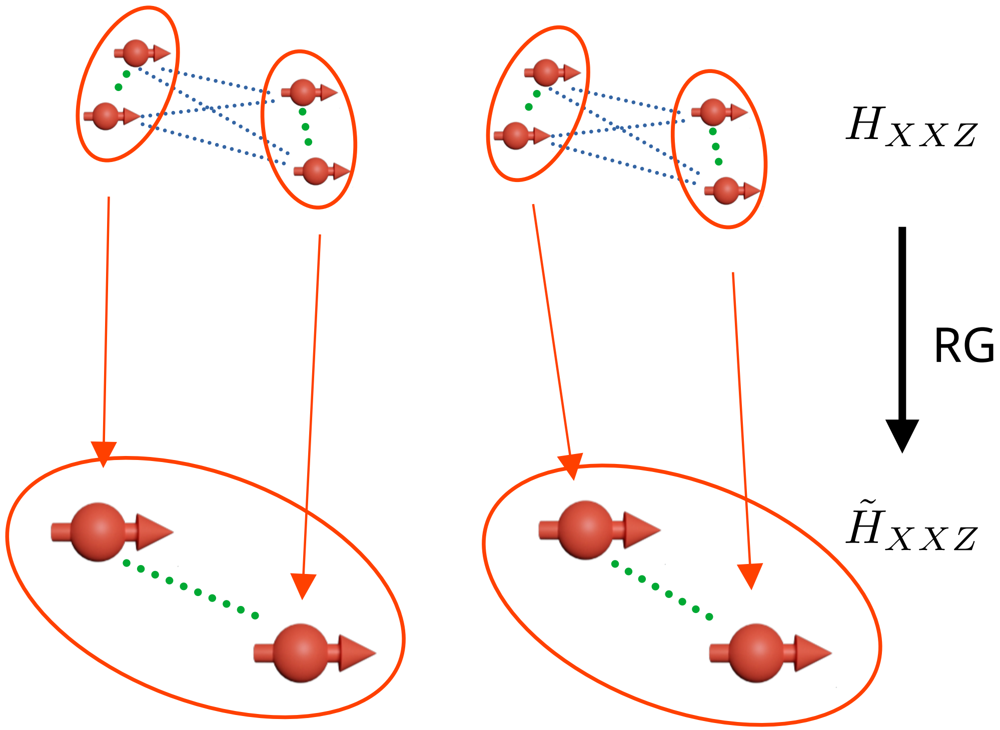
Braemer *et al.*, PRB **106**,134212 (2022)

Observation:
No drift of transition

Idea:

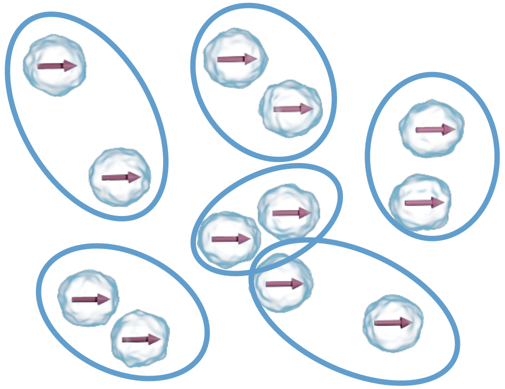


$d = 1, \alpha = 6$



Summary

$$\hat{H}_{XXZ} = \sum_{i,j} J_{ij}^{\perp} \left(\hat{s}_{+}^{(i)} \hat{s}_{-}^{(j)} + \text{h.c.} \right) + \sum_{i,j} J_{ij}^{\parallel} \hat{s}_{z}^{(i)} \hat{s}_{z}^{(j)}$$



- Disorder leads to emergent integrability
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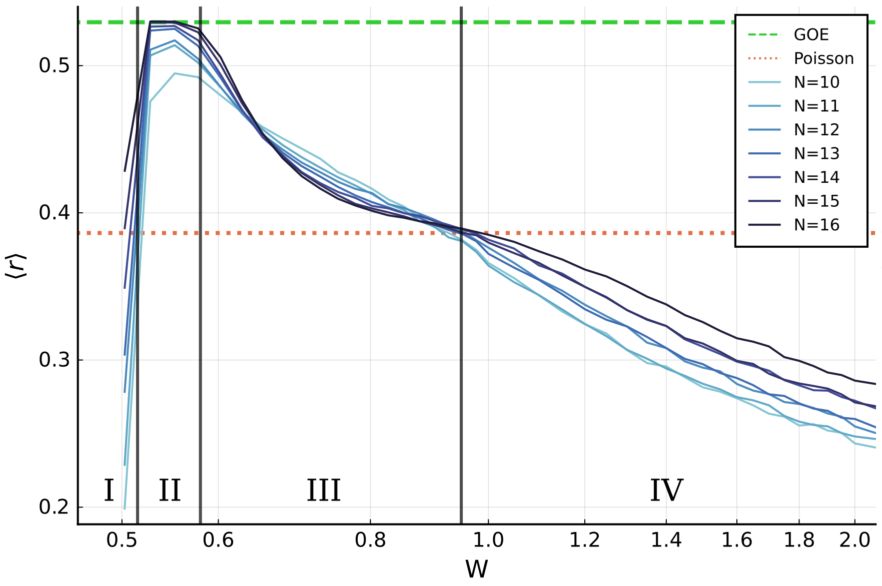

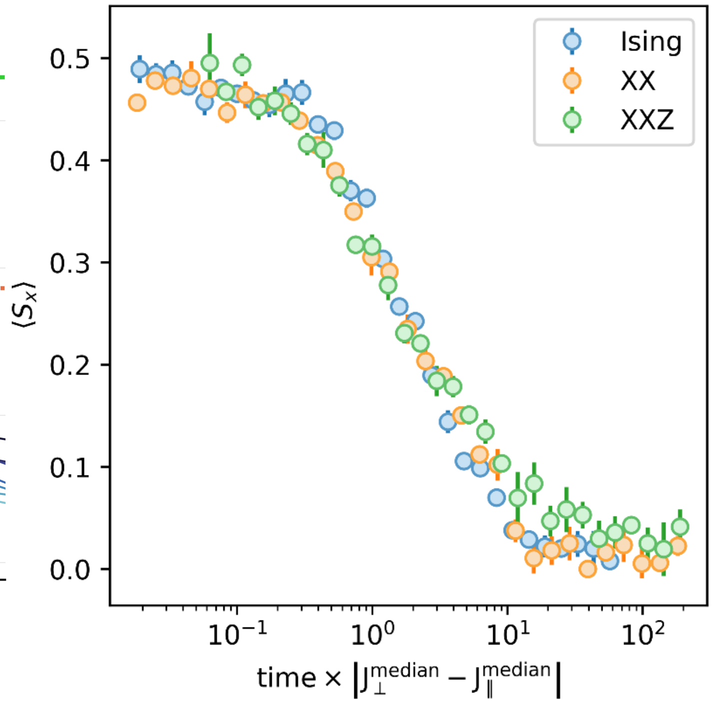

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Backup: Cusp

$$H(\Omega) = H_{XX} + \Omega \sum_i s_x^i$$

- Spin locking
- Simple model:

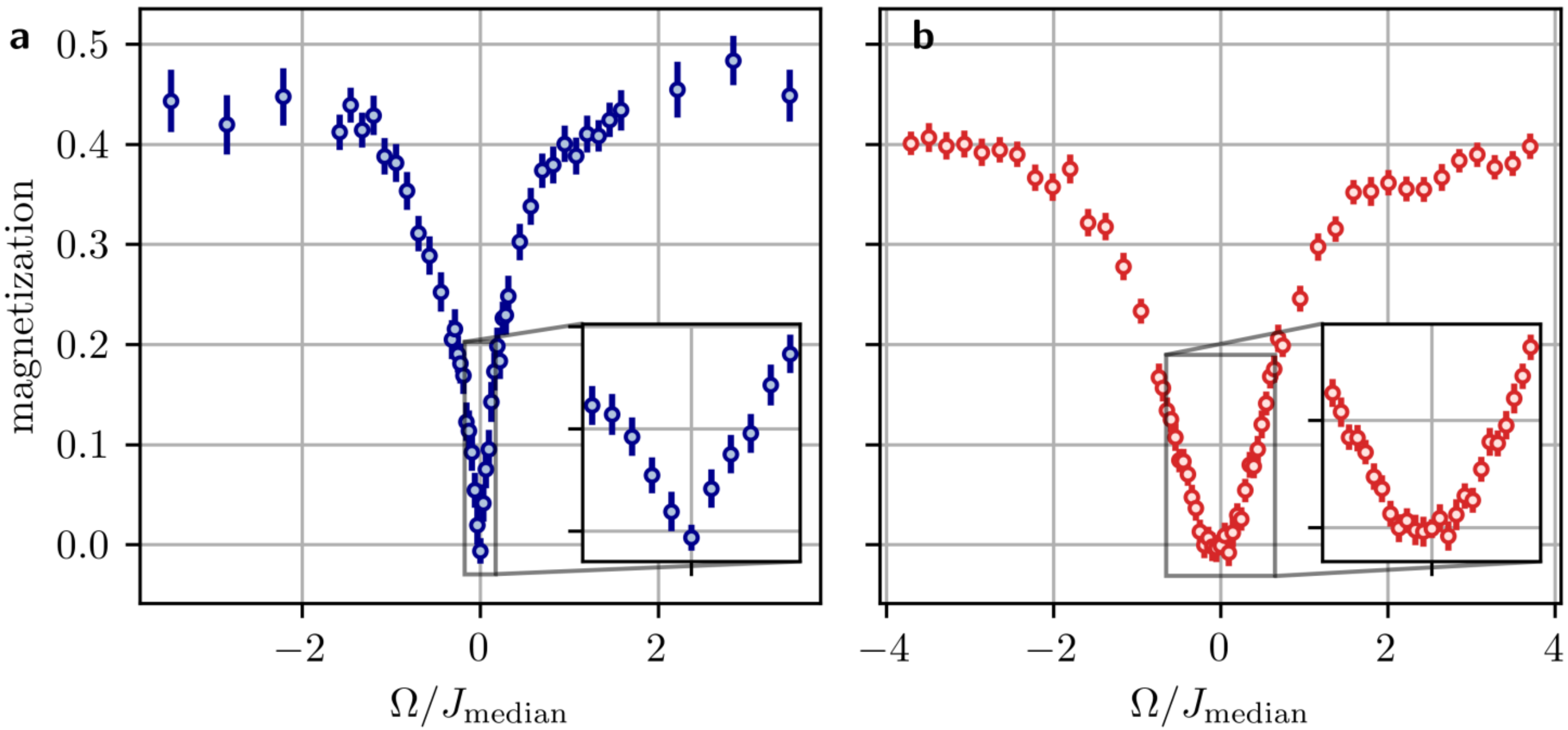
$$\langle S_x \rangle \propto \int_0^\Omega \mathrm{d}J P(J)$$

- Relevant couplings

$$P_{NN}(J) \sim J^{-\frac{d}{\alpha}-1} \exp\left(-\frac{J^{-d/\alpha}}{\lambda^d}\right)$$

$$P_{pair}(J) \sim J^{\frac{d}{\alpha}-1}$$

Steady state magnetization



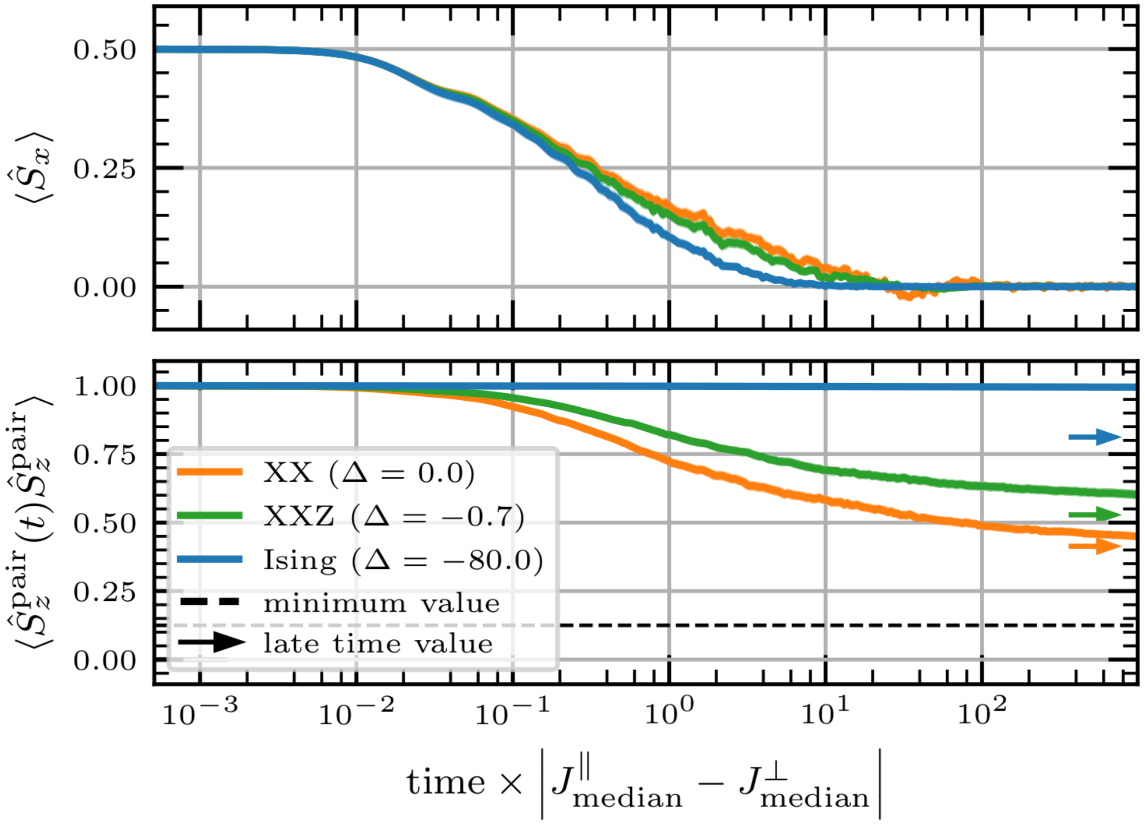
Strong disorder

Pair couplings

Weak disorder

NN couplings

Backup: Limitation of Pair Model



d=1, α=2, W=5

Pair model

$$\hat{H} \approx J_{12} \hat{H}^{(1)(2)}_{\text{pair}} + J_{34} \hat{H}^{(3)(4)}_{\text{pair}} + \tilde{\Delta}(\hat{s}_z^{(1)} + \hat{s}_z^{(2)})(\hat{s}_z^{(3)} + \hat{s}_z^{(4)})$$

Predicts conservation

$$[s_z^{(1)} + s_z^{(2)}, H] = 0$$

Prethermalization